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Research Article

An Examination of Middle School 7th Grade Students' Mathematical Abstraction Processes

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Abstract

In this study, abstraction processes of 7th grade students were examined. In addition, it has been tried to explain how the implementation of this process affects the students' academic success. For this purpose, the experiment and control groups were formed. While the current teaching program was applied to the control group, the experimental group was taught with the ACE teaching cycle which is the application dimension of the theory based on the abstraction philosophy. It can be stated that such a study is shaped according to semi-experimental method. The application was carried out in the 7th grade of a state secondary school in the province of Erzurum in the 2014-2015 academic years, with a total of 31 students in the experimental group and 32 students in the control group. Both quantitative and qualitative data were obtained in the study. The achievement tests developed by the researcher for the quantitative data and the interview forms developed by the researcher for the qualitative data were used as data collection tools. In addition, camera records and observation notes obtained from the process were used. Therefore, in this study, it was tried to provide reliability by data diversity. The analysis of the quantitative data was done by statistical tests and the analysis of the qualitative data was done by descriptive analysis method. At the end of the research, it is seen that students' abstraction level of the equation subject is better in the group that ACE teaching cycle is applied than the other group. Furthermore, it is seen that teaching in application process keeps the students' interest and motivation alive. According to the results obtained it can be said that classroom activities based on students' abstraction process may be necessary for a qualified learning.

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Introduction

Thinking is one of the characteristics given to the human beings, and it has a complicated structure. Therefore, it is not strange that mathematical thinking is to be more complicated. Therefore, Liu (2003) claimed that mathematical thinking is a system which includes complex processes such as predictability, induction, deduction, description, generalization, abstraction, exemplification and proof. These concepts play a crucial role in the development of mathematical thinking process. It is a responsibility rather than a choice to emphasize these concepts to construct students' mathematical thinking processes.

In Turkey, judgments regarding mathematical thinking started to increase particularly in recent years resulting from the changes of question types in the examination system. The studies especially carried out in the recent years with improvements in learning theories have generally been on how learning improves rather than on what level it is. It was also aimed to reveal how abstraction process that supports mathematical thinking developed in the current study. Schoenfeld (1992) stated that abstraction process should be taken into consideration for improving students' mathematical thinking, and that abstraction, symbolic representations and symbolic operations which he regarded as mathematical tools need to be adequate enough to interpret mathematical structures. This makes scrutinizing students' thinking processes necessary and important. On the other hand, it was seen that the current study's being experimental makes it significant because most of the implementations conducted via abstraction process have been qualitative (Bass & Montague, 1972; Davydov, 1990; Dienes, 1967; Dreyfus, 1991; Dubinsky, 1991; Dubinsky, Weller, McDonald & Brown, 2005; Frorer, Hazzan & Manes, 1997; Meel, 2003; Noss & Hoyles, 1996; Skemp, 1986; Sfard, 1991; Tall 1999), and that only few experimental studies have been carried out. That is, the current study would clearly contribute to the literature. Additionally, a model which was developed based on abstraction in the implementation dimension was employed in the study. Such kinds of studies are precious since they provide rich data about efficiency and productivity of the model. Bills, Dreyfus, Mason, Tsamir, Watson and Zaslavsky (2006) suggested that such kinds of studies have great impact on establishing and strengthening the models.

In the present study, an experimental implementation regarding the concept of equation with 7th graders was carried out, and it was aimed to reveal effects of this implementation. Furthermore, it was aimed to scrutinize the students' abstraction processes during the implementation. That is to say the study was based on two problems:

1. Does teaching with ACE teaching cycle conducted under the guidance of the Ministry of National Education (MoNE) affect the students' achievement levels in the topic of equation?

2. How are the students' abstraction processes in the concept of equation?

Theoretical Framework

Mathematics is a science that is based on intellectual and logical structures of individuals. Abstraction is a process which deals with individuals' mental activities like mathematics, too. This makes abstraction one of the important topics in mathematics. Thus, mathematical abstraction skills of individuals become significant.

Dienes (1967) defined abstraction as the core of common things in different situations. It can be deduced from this definition that abstraction uses a type of simplification strategy. Dubinsky (1991) stated that abstraction process consists of the steps of generalization, synthesis and abstraction. Moreover, the researcher expressed that these three steps are performed repeatedly in a cycle, and so abstraction occurs. Dubinsky (1991, 2000) emphasized five concepts within the frame of reflective abstraction process for the development of advanced mathematical thinking: interiorization, coordination, encapsulation, generalization, reversal. *Interiorization* was described as children's doing reflective abstractions to construct interior processes, which is a way of interpreting perceived phenomena with the ability of using symbols, language, pictures and mental images (Piaget, 1970a: 64; Piaget, 1980: 90). *Coordination,* on the other hand, is a combination of two or more processes for the aimed new organizations. Piaget reported that some cases need more than keeping two things in mind simultaneously. Additionally, it was considered as the coordination of two actions in one system (Bass & Montague, 1972). Whereas, existence of a dynamic process in a static object was called as *encapsulation* or *transformation*. On the other hand, *generalization* was meant by learning to practice schema formed for the subject's larger phenomena knowledge, while *reversal* was meant to give the subject the chance to think the process reversely when the process occurs interiorly. Dubinsky (1991) illustrated these mental structures of abstraction process as follows:

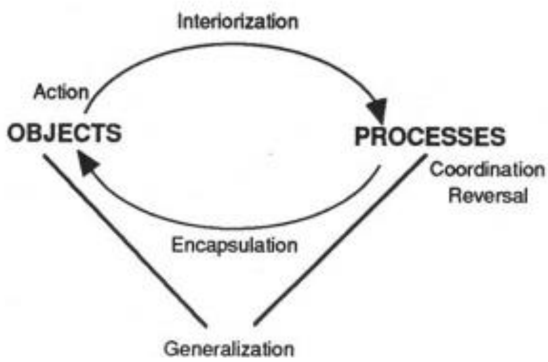


Figure 1. Structuring of abstraction process

Dubinsky constructed this structure he obtained systematically and used it as a theory. He stated that this theory arose from the fact that *'mathematical knowledge consists of three types which are actions, processes and objects. These types are organized into structures called schemas that are used for solving problems and understanding situations. Mathematical knowledge derives from individual tendency about mathematical problem situations perceived through these schemas'* (Dubinsky, 1991). This theory was named as APOS theory based on the mental structures (Action-Process-Object-Schema) mentioned in the hypothesis.

APOS theory suggests that there is a close relationship between the nature of mathematical concepts and their development in individuals' minds (Dubinsky, Weller, McDonald & Brown, 2005). Therefore, explanations about this theory do not have only epistemological but also psychological value. According to this theory, an individual deals with mathematical situations to build mental structures by using certain mental mechanisms. The mechanisms referred here are interiorization, coordination, encapsulation, generalization and reversal.

The researchers who were interested in APOS theory designed an instrument called "genetic decomposition" (Dubinsky, 1991). With the help of this tool, it is aimed to define which mental structures are built by the students (action, object, process and schema) while they are learning a concept/topic (Çetin & Top, 2014). Genetic decomposition is initially constructed theoretically. Genetic decomposition of a concept is a group of structured mental formations that can describe how the concept was developed in individual's mind (Asiala, Cottrill, Dubinsky & Schwingendorf, 1997). According to these researchers, genetic decomposition can be implemented by designing appropriate teaching and then by questioning students qualitatively. This gives the researcher the opportunity to test the designed teaching empirically. Thus they developed instructional treatments of APOS theory. In this instructional style, activities are developed to improve students' abstraction ability and to accelerate their mental structures (A); settings where students can reflect their ideas during treatment are created and they are given the opportunity to be active (C), and finally the cycle is completed by designing out-of-class activities for the students to use the concepts they learnt in the classroom and to develop the learnt concepts (E). Thus, an instructional treatment called ACE teaching cycle was developed. According to Weller, Arnon and Dubinsky (2009), this cycle is a pedagogical approach based on APOS theory.

Although there are a limited number of studies on this cycle, the studies carried out present significant data about the efficiency of this model. For instance, Çetin (2009) aimed to examine how students comprehended the topic of limit and to reveal how this comprehension would change through the designed teaching. The researcher designed teaching setting in accordance with ACE teaching cycle. It was found in that study that the designed teaching setting affected students' achievement and comprehension in the topic of limit positively. There were similar studies, too. Weller et al. (2009) carried out a study aiming to examine the preservice elementary and middle school teachers' mathematical performance on rational numbers (fraction or integer) and their decimal expansions. ACE teaching cycle was used again, and significant improvements were observed in the students' performance in that study. Tzirias (2011) also aimed to do genetic decomposition of students' understanding about the topic of ratio, and suggested that this cycle is an important tool in this topic.

Method

Research Design

The current study was quasi-experimental design, and included the experimental and control groups. In quasi-experimental design, the impact of independent variable on dependent variable is tested (Tabachnick & Fidell, 2007: 2-3). While the independent variables of the study were teaching methods applied, the dependent variables were the students' academic achievement and their skills of abstraction.

Sample

In the study, cluster sampling method was employed in choosing the experimental and control groups. In 2014-2015 academic years, two classrooms were randomly assigned as the experimental and control groups among 11 different 7th grade divisions in a state school allied to Erzurum National Education Directorate. In the experimental group, there were 31 students (15 girls and 16 boys) while there were 32 students (15 girls and 17 boys).

For the qualitative part of the research, purposeful sampling method was employed. Purposeful sampling is a qualitative sampling method which is used when detailed information about a case is needed (Yin, 2011: 88). In this method, the researcher is responsible for determining the participants that best suit aims of the subject studied. In the current study, as the students' thinking processes would be examined, the state of their representing the whole class in terms of achievement as well as their personal characteristics such as the ability of self-expression and enough self-motivation to complete the activities were taken into consideration. For the selected students, their school achievement scores, mathematics achievement scores and achievement scores received from the tests used during the implementation were taken as references. A nickname was given for each student. Individual interviews were performed with 8 students who were Fatih, Ezgi, İslim and Sena in the experimental group and who were Talha, Okan, Hakan and Nur in the control group.

Data Collection Tools

As the data obtained in this study were both quantitative and qualitative, separate instruments were developed for both methods. Two algebraic teaching tests were developed to gauge possible effect of implementation dimension of the study by the researchers. The first of these tests was developed by the researcher to test prior knowledge of the students about the topic on which they were required to abstract. Thus, this test contained 6 algebraic learning outcomes taught prior to sub learning domains of equations. The researcher created a question pool including 60 questions regarding 6 attainments equally. Thus, it was aimed to provide content validity of the test and a draft form consisting of 30 questions was prepared. Each item in the draft form was asked for the opinions of experts who were 1 faculty member, 2 research assistants and 2 mathematics teachers. Furthermore, test items were scrutinized by a Turkish language teacher in terms of linguistic appropriateness. The experts were asked to give feedbacks for each item as "suitable/not suitable". The draft form was revised and finalized. This pre-test was applied to 167 7th graders, who were familiar to the research subject, in the second week of May in 2013-2014 academic year - spring term. Item analysis was made for reliability of the test following content validity and expert opinion. At the end of the analysis, the test which aimed to gauge the students' levels of prior knowledge and which contained 20 multiple-choice items was created (Pre-test). Cronbach Alpha reliability coefficient of the test was found .86. On the other hand, the second test covered 5 learning attainments related to sub learning domain of equations. A similar process like pre-test was applied for this test, and firstly a question pool including 50 questions was developed. Then a draft form consisting of 20 questions chosen from this pool was obtained. After required reviews, the draft form was applied to 158 7th graders.

Following the item analysis, the test which consisted of 15 multiple-choice questions and which was for measuring the students' levels of knowledge on the topic of equation was shaped (Post-test). Cronbach Alpha reliability coefficient was found to be .82.

On the other hand, a semi-structured interview form was developed by the researcher to reveal the impact of the implementation in more details. Within the frame of form implementation attainments, it consisted of 3 problem situations aiming to reveal the students' characteristics of organizing, arranging, applying and reflecting the knowledge related to real life circumstances they learnt to new circumstances. In developing this form, the draft form consisting of different problem situations was applied to 31 7th graders, and then interviews were made with 3 students selected among these students. As a result of analysis, 3 of these problem situations were decided, and some statements were edited. The first problem situation was a company scenario related to analysis of abstraction process of the students on creating equation using algebraic concepts like variable, equilibrium and pattern. The second problem situation was a construction scenario measuring the students' ability to interpret and to solve the equation given. Finally, the third problem situation was about the problem scenario which gave the students the opportunity to use the concepts they learnt and which they created. Pilot study of the form was performed, and time to be allocated for interview was determined. The worksheets obtained from the students during these interviews were among data collection tools.

Implementation Process

The implementation was carried out during 20 lesson hours for both groups between 03 November 2014 and 28 November 2014. Each lesson lasted for 40 minutes. The details of the work schedule are presented in Table 1.

Table 1. Learning outcomes of the implementation and number of lesson hours

Learning Outcomes	Date	Time Allocation
Students are able to do addition and subtraction with algebraic expressions.	03-07 November	3 lesson hours
They are able to multiply two algebraic expressions.	03-07 November	2 lesson hours
Implementation	10-14 November	1 lesson hour
They are able to express relation in number patterns in letters by modeling these patterns.	10-14 November	2 lesson hours
They are able to solve simple equations with one unknown variable.	10-14 November	3 lesson hours
Implementation	17-21 November	2 lesson hours
They are able to use equations in solving problems.	17-21 November	3 lesson hours

They are able to explain linear equations.	24-28 November	2 lesson hours
Implementation	24-28 November	2 lesson hours

The implementation process in the experimental group was conducted through ACE teaching cycle. Firstly, 19 activities related to the learning outcomes were created by the researcher in order to practice this process. The pilot study was carried out with some of the activities created to keep possible negative effects to be experienced during the process under control and to gain experience. Various sources were used in preparation of the activities (Abels, de Jong, Dekker, Meyer, Shew, Burrill & Simon, 2006; Kindt, Roodhardt, Wijers, Dekker, Spence, Simon, Pligge & Burrill, 2006; Kindt, Wijers, Spence, Brinker, Pligge, Burrill & Burrill, 2006; Kindt, Dekker & Burrill, 2006; Wijers, Roodhardt, van Reeuwijk, Dekker, Burrill, Cole & Pligge, 2006). The implementation process was recorded with a camera set up to see the whole class. The recordings were started three weeks before the real implementation to make the students familiar to the camera. The students' active participation was encouraged during the activities, and great effort was expended to create a setting in which they could express their thoughts easily.

Contrarily, the implementation process of the control group was shaped according to the book proposed by MoNE (2014). That is to say, the instructions about the sub learning domain in the source book were performed without any changes. Techniques such as active learning, question-answer, brainstorming, discussion and research were used in the learning outcomes approved by MoNE (2014). Apart from these, both individual activities and group activities were performed with the students.

Data Collection Process and Data Analysis

The pre-test before the intervention and the post-test following the intervention were conducted on both of the groups simultaneously in a lesson hour. The data obtained from these tests were analyzed using some statistical techniques with SPSS-22 program. As it was aimed to reveal possible effects of the instruction implemented in the experimental and control groups on the students' academic achievement in this study, that whether two independent variables (teachings performed) caused any significant differences on a dependent variable (achievement level) was investigated. This case required the analysis of independent groups t-test. As a result of the study, the data obtained were evaluated on .05 significance level.

The implementation lasted for 20 lesson hours, and then interviews were performed separately with eight students selected through convenient sampling method. The interviews were recorded through a camera which just focused on the worksheets on which the students wrote their thoughts. The students were given a worksheet which was written on their scenarios during the interviews, and they were asked to reflect their opinions on these sheets. Thus, the written documents were obtained from them. Materials to be used by the students such as graph papers, squared papers, ruler and colored pencils were kept available during the interviews, and they were given to the students upon their request. Each interview lasted for approximately 50 minutes. The interview records were transmitted into written documents by being transcribed. The students' non-verbal communication was observed as well as the verbal and written data. The obtained observations were included in the findings of the interview texts.

The interview texts and the student worksheets were analyzed via descriptive analysis method. Descriptive analysis is summarizing and interpreting the data according to the predetermined categories (Yıldırım & Şimşek, 2011: 224). The data obtained in the research were scrutinized based on mental mechanisms of abstraction process. With reference to the definitions of these mechanisms, key words for each mechanism were created (Table 2). Briefly, behaviors in this table were mostly applied in determining which mechanisms were used by the students.

Table 2. Key words related to mental mechanisms

Interiorization	**Coordination**	**Encapsulation**	**Generalization**	**Reversal**
• Comparison • Reflection • Realization • Interior description • Description	• Integration • Composition • Keeping something in mind together	• Synthesizing • Protection	• Further practicability • Awareness about practicability of the created structure • Association • Investigation (Associating similarities)	• Recursion

Findings

In this section, the findings regarding quantitative and qualitative data of the research were presented under separate headings. While the data related to analyses showing development within and between the groups were presented under the heading of '*the findings regarding quantitative data*', the interviews performed with the students in the experimental and control groups at the end of the intervention were presented under the heading of '*the findings regarding qualitative data*'.

The Findings Regarding and Interpretation Quantitative Data

It was aimed to gauge the students' prior knowledge about the research topic and to compare similarities of the groups in terms of pre-requirements. For this aim, the pre-test developed by the researcher was conducted on both groups in a lesson hour simultaneously just before the intervention. Analyses related to this application were given in the following table.

Table 3. Independent groups t-test results regarding pre-test mean scores of the students in the experimental and control groups

Groups	N	$\bar{X}$	SD	df	F	t	p
Experimental Group	31	45.16	27.55	56.35	4.63	.78	.43
Control Group	32	40.31	21.21				

In Table 3, it was seen that there were not any significant differences between pre-test mean scores of the experimental and control groups ($t(56.35) = .43; p > .05$). Thus, it can be claimed that the groups were equal in terms of pre-requirements of the intervention.

The intervention was started after it was understood that the groups were similar. The students' achievement levels were measured with post-test after teaching in the experimental and control groups. The results regarding this test were presented in Table 4.

Table 4. Independent groups t-test results regarding post-test mean scores of the students in the experimental and control groups

Groups	N	$\bar{X}$	SD	df	F	t	p
Experimental Group	31	75.61	20.10	61	.57	4.29	.00
Control Group	32	54.90	18.03				

According to the data in Table 4, a statistically significant difference was found between mean scores of the research groups ($t(61) = .00; p < .05$). The difference was in favor of the experimental group because post-test mean scores of the experimental group ($\bar{X} = 75.61$) were higher than post-test mean scores of the control group ($\bar{X} = 54.90$). Although the control group had a mediocre achievement, the achievement levels of the experimental group students were fairly better than of the control group students.

The Findings and Interpretation Regarding Qualitative Data

In this section, the data of the interviews which were performed with the students in the experimental and control groups at the end of the intervention were explained under separate headings. 3 problem situations were included in the interview form. The first one was about revealing, showing and interpreting the association between the number of bulbs sold in a company and the profit obtained. The second was a scenario of construction which was about revealing, showing and interpreting the association between the number of sacks carried and the elapsed time. On the other hand, the third one was about the students' skills to create a problem situation for which they would use equations.

The Data of Interviews Obtained from the Experimental Group and Their Interpretation

It was observed that the students in the experimental group were generally able to relate the problem situations they encountered, and they could adapt them to new situations by describing the necessary concepts accurately. Additionally, it can be stated that they checked the data they obtained by trial and consolidated the structure they created in their minds through different representations. For instance, a part of the data belonging to Fatih related to the first problem situation was as follows:

…

12F: Yeah, suppose that a man buys two bulbs, that is, the items sold are 2. If he buys 2 the price is 16, that is, he will make a profit of 4 liras on the whole. If he raises this (showing a), this (showing k) will increase evenly. If he bought 4 items, the profit earned would be 8 liras. That is to say, it is twofold.

13A: Can you express in more details what you mean by twofold?

…

16F: Teacher, I guess I made a mistake. If the purchase price is 8 liras, and if we sell it at a profit of 25%, the selling price will be 10 liras (He just explains this orally without using a pencil or paper)… 25% means 1/4, and ¼ of 8 liras is 2. We can find that result is 10 liras when we sum up.

17A: … All right. What can you say for the relation between a *and* k*?*

18F: I say that it is directly proportional. That is, it always doubles. There are no other patterns, so it is directly proportional.

19A: Can you explain this relation in one sentence by considering a *and* k*?*

20F: Since k *is twice* a*, it is directly proportional.*

When the conversation text above was analyzed, it was found that the relation between the items sold and the profit made took shape in Fatih's mind (12F, 20F). Furthermore, the fact that he tried to interpret the question by using concepts such as fractions corresponding to percentile, direct proportion and pattern was a sign for his benefiting from his prior knowledge while creating the new knowledge (16F, 18F). That is to say, Fatih could exactly comprehend what the variables mean and associate them. Thus, he could reflect the knowledge he had obtained before to new situations through interiorization. In this respect, it can be thought that he behaved at the process stage.

Another prominent point in the interviews carried out with the students in the experimental group was that they firstly tried to make sense of the problem situations they encountered in their own mind and meanwhile they tried to prompt their prior knowledge. For instance, this was clearly stated in the interview performed with İslim.

10İ: Let's start by finding the selling price first. 25% of 8 is its ¼… That is, it is sold for 10 liras which is found by $8+2$ *. We say* a *for the items sold, and* k *for the profit obtained. For example, number of sold items is* a*, and a profit of 2 liras is earned when 1 item is sold. When 2 items are sold, a profit of 4 liras is gained. Thus, we will do like so (suddenly gets quiet). What if we do in table (immediately begins to draw a table).*

11A: Then, do it.

12İ: a*, what is* a*,* a *is the items sold and* k *is profit. When 1 item is sold, a profit of 2 liras is gained. When 2 items are sold, a profit of 4 liras is earned, 6 liras of profit for 3 items and 8 liras of profit for 4 items. That is, it is directly proportional.*

When the conversation above is analyzed, it can be seen that the student firstly tried to interpret and to identify the problem situations (10İ, 12İ). Moreover, it is remarkable that İslim had the knowledge of expressing the relation she realized with suitable concept (12İ). It was similar in the data obtained from the second scenario of the form of interview performed with Sena.

…

45S: $a = 12.(4 - t)$, that is, a is the number of sacks carried. If you multiply 12 by 4, subtract the elapsed time from this number, and multiply this by 12, you can find the number of sacks.

46A: Yeah.

47S: We should find the elapsed time when Kemal started to work. A sack of sand to the second floor (reading the rest silently). Now, if we find t time when it was not carried, we can find the number of sacks not carried as well. How many sacks, we say x for the number of sacks, and we subtract a, the number of sacks not carried, from x. Then we can find the number of sacks filled with sand when Kemal started to work. Now, well, a here is the number of sacks not carried, and it is time here (starts to think silently).

…

In the beginning, Sena tried to identify what a and t mean and to interpret what each term of the equation represented (45S, 47S). It is understood from these expressions that Sena preferred to think alone rather than wait for help. This is a sign for her exerting effort to cope with the problem situation she encountered.

Another point to be emphasized in the experimental group was the students' having worries of generalization. Even though Sena was regarded as a less successful student, her expressions in the stage of creating a problem were quite remarkable. In the worksheet below, the problem situation created by Sena and her solutions related to this problem were given (Sena's expressions are translated into English without changing). Then, the student's verbal statements were included.

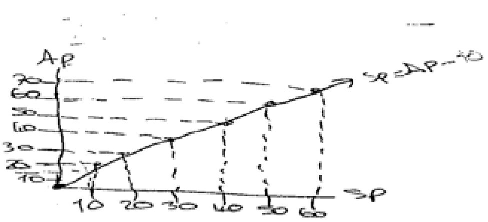
1) *Ayşe has a maths exam. Sertaç's score in this exam is 10 points lower than her. Then, create a graphic and a table illustrating Sertaç's score.*

   Sp= Ap-10 *Sp: Sertaç's score*

   Ap: Ayşe's score

2) *Create a table and a graphic by associating Sp and Ap.*

Ap=10 ise Sp=0
Ap=20 ise Sp=10
Ap=30 ise Sp=20
Ap=40 ise Sp=30



Ap	10	20	30	40
Sp	0	10	20	30

Figure 2. Sena's worksheet

I thought two different people and evaluated their scores according to each other. Thus, I considered the relation between two unknown… Then, I wrote my equations. I calculated mathematics scores of the two students based on these equations. As the lowest score would be 0, I began from 10… I created a table and a graphic according to these equations, and I associated Ayşe's and Sertaç's scores. However, their scores are not certain, anyway I did not question this. I just tried to show in a graphic and in a table…

Sena, tried to create data of two different people to use her knowledge of variables, and defined each person as a variable. Upon creating the equation revealing the association between two variables, she calculated which values these variables could take. It was observed that the student did not consider determining limit superior while determining limit inferior of the variables. Sena's views above made us think that she regarded equation as an object showing the association between variables. Then, her purpose in creating a graphic and a table was to support the association she mentioned with visual data.

The Data of Interviews Obtained from the Control Group and their Interpretation

The ability of the students in the experimental group to make sense and to describe the problem they realized was not observed in the students in the control group. The students in the control group generally tried to solve the question and waited for the explicit and clear solution. They mostly stated that the things given were inadequate. It can be seen clearly in the data of the interview performed with Okan.

…

47O: Well, the number of sacks? Teacher, I think we cannot find.

48A: Why not?

49O: Because, we need one more known fact… that is, number of the sacks not carried, carried or the rest sacks should have been given.

…

Like in the interview performed with Okan, the same situation was seen in other members of the control group. Firstly, the students told that there was missing data in the question, and that it was impossible to solve it. However, they could make some interpretations with the help of clues given by the researcher, and they could get ahead. This fact indicated that they needed an exterior support. The analysis related to Harun's data is as follows;

…

55H: Now, we will find the number of sacks filled with sand. Actually, we cannot find an exact number since no values are given, and we have to give a value.

…

61H: Kemal started to work with how many sacks (starts to think silently), I think we cannot find teacher. We can just find that part by giving a value.

…

66A: All right, can you find the number of sacks of sand available when he started to work?

67H: When he started to work? (becomes silent). Then, should we give a value for a?... in fact I've found the number of sacks, (by muttering) how many sacks of sand are there when he started to work? If he completes in 4 hours… Teacher, if t is 0, hmm, when we give the value of 0 to t, we can find the number of all sacks.

…

69H: For example, here 0 for t… (performs necessary mathematical operations) it is 48, I mean, a is 48 before he started to work.

Harun claimed that he could not find a clear solution as there were not any clear values in the question (55H). The same situation was observed in Harun's expression of 61H. Harun could find with how many sacks Kemal started to work and how many hours it took to finish work. However, he told that he could find this result just because of his giving values, and actually it was not possible to calculate this as no values were given (55H, 61H). Then, he could interpret the variables (67H, 69H) with the help of a small hint provided by the researcher (66A). This proved that Harun needed an exterior hint in the process of forming knowledge. Another remarkable point of the data was the fact that the students in the control group mostly did not pay attention to their mistakes and continued to make same mistakes. A part of the worksheet in which Harun's expressions about the second scenario were included was presented below.

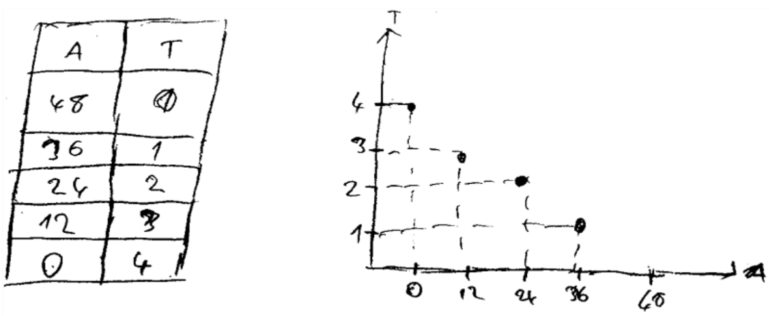


Figure 3. Harun's worksheet

Let's pay attention to the position of 0 in Figure 3. As a result of the interview performed with Harun, it was seen that he associated the variables accurately and tried to explain this association with the help of a table and a graphic. Harun could not develop the graphic to reflect his expressions whereas he created the table accurately with the values he found. Additionally, he stated that verbal expressions like not starting to work and carrying all sacks were associated with 0, but he could not show this on the graphic. In this section, the researcher expected the student to show diminishing-way relation between the variables on the graphic, and to realize he created incorrectly. However, Harun could not achieve this exactly (Şekil 3).

The students in the control group particularly avoided from using algebraic statements in the final scenario in which they were expected to develop a problem situation that would help their using equations. It was found that the problems they created generally had a direct solution, had clear data and their unknown values would be found easily. The worksheet related to Talha's data is as follows (Talha's expressions are translated into English without changing):

A wheel of a car wears out 2 cm per kilometer. Explain the wheel's wearing out per kilometer in a table and in a graphic. (Width of the car wheel is 30 cm.)

Beginning *Per kilometer*

30 cm	*28*	*26*	*24*	*22*

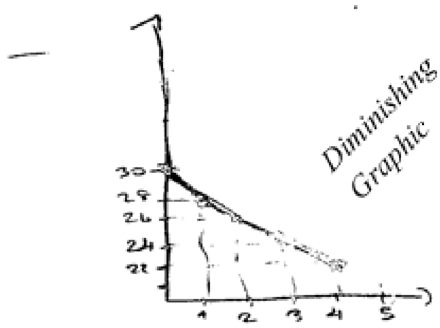


Figure 4. Talha's worksheet

As seen in Figure 4, Talha directly answered the question and showed it through a table and a graphic. Hence, it can be claimed that he did not need to use his knowledge of variables and of equation. Quality of the question or finding a solution for it was not a point examined in the research. The purpose here was wondering about how the student would reflect the things he learnt during the process by himself. Talha found new size of the wheel per 4 kilometers above. Firstly he found the solution by making a table, and he said he could find the solution more easily by making table. This was a sign for the student's being solution-oriented, so not being focused on the relation between the variables.

Finally, it was noticed that the students in the control group frequently used association like the ones in the experimental group did. It was also found that they preferred to adapt new concepts they previously learnt to new situations and paid attention to benefit from various demonstrations. However, it was understood from the findings above that they mostly preferred to be superficial. This is one of the most basic and so of the biggest hinders for abstraction.

Discussion and Conclusions

In this section, the findings were separately discussed based on their being quantitative or qualitative. While aim of the quantitative findings was to reveal effectiveness of instructional intervention, aim of the qualitative findings was to reveal possible impact of instructional intervention on the students' abstraction process.

Although no significant differences were observed between the students according to the results of the pre-test which was conducted prior to intervention, a significant difference ($p < .05$) was found between post-test mean scores of the students in the experimental and in the control groups in favor of the experimental group at the end of the implementation. With reference to these results, it can be suggested that teaching designed based on ACE teaching cycle affected the students' academic achievement in the topic of equation positively. Following the intervention, achievement mean scores of the experimental and control groups were $\bar{X} = 75.61\ ve\ \bar{X} = 54.90$ respectively. Thus, it can be claimed that achievement scores of both of the groups were medium-level and above. There are two prominent points here:

(1) It was noticed that the students' post-test achievement scores were higher than their readiness achievement scores before intervention. In other words, it can be stated that teaching performed for the two groups was successful in itself.

(2) The differentiation at the end of the intervention was in favor of the experimental group, and the more significant issue was that teaching based on ACE teaching cycle was more effective on the students' achievement scores in the topic of equation than teaching performed under the guidance of MoNE.

According to Dubinsky (2000), APOS theory or ACE teaching cycle which is derived from this theory can be an appropriate tool defining student development in learning mathematics. There are several researchers who adapted this teaching cycle into their teaching plans and investigated effects of it on students' thinking processes (Asiala vd, 1997; Asiala, Dubinsky, Mathews, Morics & Oktaç, 1997; Cottrill, Dubinsky, Nichols, Schwingendorf, Thomas & Vidakovic, 1996; Çetin, 2009; Çetin & Top, 2014; Kathleen, 1999; Maharaj, 2013; Murray, 2002; Tzirias, 2011; Weller, Arnon & Dubinsky, 2009; 2011). These are the studies revealing positive results of teaching based on ACE teaching cycle.

It is as difficult to claim that a model is efficient just by checking the quantitative data as asserting that it is ineffective (Çetin & Top, 2014). Therefore, suggesting that a model is efficient just based on quantitative data can be inadequate. In this respect, it is crucial that efficiency of this model was highlighted again in the qualitative results of the current research.

It can be argued that some important points stood out in the light of the detailed evaluation of the abstraction process. The first was that the students' process of abstracting a concept was versatile. While these choices helped some students reach the ultimate goal, they were not useful for the others. In the current study, that some students made progress more easily by using visual shapes while some others reached the same point just by using algebraic expressions is an example for this. This result is consistent with results of some studies (Özmantar & Monaghan, 2007; Sezgin Memnun, 2011).

Another point arouse in the interviews performed was the students' tendency to prefer notations they were previously familiar in using variables. Some students in the experimental and control groups expressed the variables in the scenario given with different notations such as x and n, and then they considered to change them within the frame of the scenario. In this regard, the students mucked the existing situation up. Kinzel (2001) concluded that the students were insufficient in interpreting and using notations they were less familiar in the perspective of algebraic learning. This result of that research shows similarities to the results of studies carried out by Soylu (2008), Knuth et al.(2005) and Zazkis and Liljedahl (2002).

Another point regarded as significant in abstraction process was the difference existing in the experimental and control group students' levels of interpreting algebraic structures. It was seen in the analyses that the students in the control group were not able to use necessary cognitive processes efficiently and exactly while abstracting the topic of equation. This showed that the students in the control group were more insufficient in comprehending algebraic structures than the ones in the experimental group. Furthermore, these students' not being able to focus on relations among numbers, to coordinate small information units for creating broader information and to think integrally resulted in their not being able to construct the structures included in the abstraction process. From this aspect, probably the basic step of students' being able to abstract a concept is associated with their sufficiency in understanding the related concept. The studies carried out by Kieran (2004) and Garcia Cruz and Martinon (1998) supported this result.

Another result obtained from the abstraction process was positive effects of using visual shapes in this process. It was seen that the students in the experimental group who tried to abstract the concept of equation could interpret and generalize relations among numbers just after shapes such as graphic and table. It was also found that the students in the experimental group who were trying to abstract the concept of equation were only able to interpret and generalize relations among numbers after shapes like graphics and tables. Additionally, that the students wanted to deal with the information units which they could not interpret after using visuals showed that it motivated the students positively. However, it was realized that the students in the control group just used these visuals, and they did not advance a step further. Therefore, it can be suggested that using these visuals in abstraction process is not adequate enough, but it helped the students create the related concept. This result of the current research is similar to the results of some previous studies conducted on importance of visuals in abstraction process (Çetin & Top, 2014; Kabael & Tanışlı, 2010; Yılmaz, 2011).

It was revealed in the current study that all of the students benefited from tables and graphics for solving the problem situations they created; however, this process showed differences in the experimental and control groups. In the experimental group, the process was generally transforming patterns into number patterns by creating tables, representing association between the variables through algebraic expressions and illustrating variation of dependent variable by independent variable on graphics. On the other hand, the process in the control group was generally showing patterns with numeric data in tables, illustrating associations between the variables on graphics without mentioning knowledge of variable and equation. Thus, it can be claimed that the students in the control group did not focus on association between variables. Furthermore, the effort made to express general term algebraically with reference to numeric patterns by the students in the experimental group is indicative of the term "generalization". Briefly, it can be stated that the students who did not focus on associations between the variables did not consider showing hypotheses they obtained algebraically, that is to say, they did not prefer generalization. This result is similar to the result of the study carried out by Yeşildere and Akkoç (2011).

Another remarkable result was about the concepts related to the topic of equation used in the students' explanations. It can be claimed that awareness levels of the experimental group students were better than of the control group students in terms of their using concepts such as variable, equation and pattern in their explanations, and of their using terms like rising graphic, decreasing graphic and linear relationship in their graphic drawings. This result of the study shows similarity with the results of some other studies carried out by Asiala et al. (1997), Wachira, Roland and Skitzki (2013) and Cooley (2002).

Suggestions

The study was implemented empirically, and individual interviews were performed with the students determined via purposeful sampling method. It was concluded that teaching in the experimental group supported the students' using the concepts related to the topic of equation accurately. Thus, possible effects of this teaching cycle on eliminating misconceptions can be investigated in detail. Finally, an analysis of development stages of a concept in students' minds can provide advantages particularly for the topics which are difficult to be understood. Teachers are highly recommended to examine such studies and to benefit from implementations performed.

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