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On the Role of 'Reciprocal' of the Strong Coupling Constant in Nuclear Structure

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ABSTRACT

Considering 'reciprocal' of the strong coupling constant as an index of strength of nuclear binding energy, it was reviewed the basics of nuclear binding energy and nuclear stability.

Keywords: Beta stability line, Semi-empirical mass formula (SEMF), Fermi gas model, Strong coupling constant

1. Introduction

According to semi empirical mass formula (SEMF) [1,2,3]: 1) There exist 5 different energy terms and 5 different energy coefficients. 2) The average binding energy per nucleon is approximately 8 MeV, whereas the maximum binding energy per nucleon is around 8.8 MeV. According to the Fermi gas model (FGM) [4], nucleon's mean kinetic energy is around 20 MeV. In a unified approach, in this paper, we reviewed our views [5-9] on nuclear binding energy with respect to nucleons mass difference, beta stability line, SEMF, FGM and strong coupling constant $\alpha_s \cong 0.1186$ [10]. Our model [9,10] is under development, it will be very simple and realistic compared to the new integrated model proposed by N. Ghahramany et al. [11-13] .

2. About the semi empirical mass formula

Let A be the total number of nucleons, Z the number of protons and N the number of neutrons. According to the semi-empirical mass formula, nuclear binding energy:

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm \frac{a_p}{\sqrt{A}} \quad (1)$$

Here a_v = volume energy coefficient, a_s is the surface energy coefficient, a_c is the coulomb energy coefficient, a_a is the asymmetry energy coefficient and a_p is the pairing energy coefficient. If we consider the sum of the volume energy, surface energy, coulomb energy, asymmetry energy and pairing energy, then the picture of a nucleus as a drop of incompressible liquid roughly accounts for the observed variation of binding energy of the nucleus.

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By maximizing $B(A,Z)$ with respect to Z, one can find the number of protons Z of the stable nucleus of atomic weight A as,

$$Z \approx \frac{A}{2+(a_c/2a_a)A^{2/3}} \text{ and } A-2Z \approx \frac{0.4A^2}{A+200} \qquad (2)$$

By substituting the above value of Z back into B one obtains the binding energy as a function of the atomic weight, $B(A)$. Maximizing $B(A)/A$ with respect to A gives the nucleus which is most strongly bound or most stable.

3. New concepts and semi empirical relations of nuclear binding energy and stability

It is suggested that:

1) Nuclear binding energy can be understood by a single energy coefficient, which is assumed to be associated with 'reciprocal' of the strong coupling constant.
2) The strength of nuclear binding energy seems to start from $(5.4 \text{ to } 6.3)$ at $Z=2$ and reaches to $(1/\alpha_s) \cong 8.432$ in between $Z = 25$ to 30. From there onwards it remains constant. Thus, $Z \approx (25 \text{ to } 30)$ can be considered as a characteristic reference zone and reciprocal of the strong coupling constant [5,8] can be considered as an index of strength of nuclear binding energy.
3) Both the strong coupling constant and the exponent of the ratio of nucleon mass difference to electron mass, play a crucial role in understanding beta stability line and corresponding nuclear binding energy.
4) For $(Z > 30)$ and $(R_0/2) \cong 0.62$ fm, $\left(\frac{1}{\alpha_s}\right)\frac{Ze^2}{4\pi\varepsilon_0 (R_0/2)} \cong Z \times 19.6$ MeV seems to represent the nuclear binding energy of stable mass number close to beta stability line.
5) Based on mass numbers, for $R_0 \cong 1.265$ fm, characteristic nuclear binding energy potential is around $\left(\frac{1}{\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_0} \cong 9.6$ MeV. On counteracting the coulombic interaction, effective nuclear binding energy potential seems to approach, $\left\{\left[\left(\frac{1}{\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_0}\right] - a_c\right\} \cong B_0 \cong 8.9$ MeV.
6) Binding energy of stable isotope near to beta stability line can be considered as a reference for estimating the binding energy of other stable and unstable isotopes of Z.
7) The ratio of neutron number to stable neutron number seems to play a crucial role in estimating the binding energy of corresponding neutron number.
8) For $(Z > 30)$, close to the beta stability line, nuclear binding energy and Z protons kinetic energy seem be approximately equal in magnitude [6].
9) In deuteron, there is no strong interaction between neutron and proton.

4. Beta stability line with respect to strong coupling constant

If $\alpha_s \cong 0.1186$, close to the line of beta stability,

$$A_s \cong \left(Z + \sqrt{\frac{1}{\alpha_s}}\right)^{6/5} \cong (Z+2.904)^{1.2} \qquad (3)$$

$$\left.\begin{aligned} (A_s - 2Z) &\cong (Z\beta + 1)^2 - 4 \\ \text{where } \beta &= \left(\frac{3}{5}\right)\alpha_s. \end{aligned}\right\} \qquad (4)$$

5. Beta stability line with respect to nucleon mass difference

With reference to nucleon and electron rest masses [10], we noticed that,

$$\exp\left(\frac{(m_n - m_p)c^2}{m_e c^2}\right) \cong 12.5659102 \cong 4\pi \qquad (5)$$

$$\left\{\begin{aligned} &\text{where, } m_n c^2 \cong 939.565413 \text{ MeV}, \\ &m_p c^2 \cong 938.272081 \text{ MeV};\ m_e c^2 \cong 0.5109989461 \text{ MeV} \end{aligned}\right.$$

Based on this observation, beta stability line can be understood with the following empirical relations.

$$\text{Let, } k \cong (1/4\pi)^2 \cong 0.006333 \qquad (6)$$

$$\left.\begin{aligned} A_s &\cong 2Z + (1/4\pi)^2 \cong 2Z + 0.006333Z^2 \cong 2Z + kZ^2 \\ N_s &\cong Z + (1/4\pi)^2 \cong Z + 0.006333Z^2 \cong Z + kZ^2 \\ A_s - 2Z &\cong (1/4\pi)^2 \cong kZ^2 \text{ and } \frac{Z}{\sqrt{A_s - 2Z}} \cong 4\pi \end{aligned}\right\} \qquad (7)$$

Based on these relations,

$$\left.\begin{aligned} &\text{A) } \frac{(A_s - 2Z)^2}{A_s} \cong k^2 A_s (A_s - Z)\sqrt{Z} \\ &\text{B) } \frac{A_s^{1/2}(A_s - Z)^{1/4} Z^{1/8}}{\sqrt{A_s - 2Z}} \approx \frac{1}{\sqrt{k}} \approx 4\pi \end{aligned}\right\} \qquad (8)$$

6. Nuclear binding energy with respect to proton number and strong coupling constant

Case-1: With reference to our recent publications [11,12], close to the beta stability line, it is possible to show that, For $(Z > 30)$, If $\alpha_s \cong 0.1186$ and $R_0 \cong 1.24$fm,

$$\left.\begin{aligned}(B)_{A_s} &\cong \left(\frac{1}{\alpha_s}\right)\frac{2Ze^2}{4\pi\varepsilon_0 R_0} \cong \left(\frac{1}{\alpha_s}\right)\frac{Ze^2}{4\pi\varepsilon_0 (R_0/2)} \\ &\cong Z\times\left(\frac{2.32253\ \text{MeV}}{\alpha_s}\right) \cong Z\times 19.6\ \text{MeV}\end{aligned}\right\} \quad (9)$$

$$\text{and}\ \begin{cases} A_s \approx \left(\dfrac{(B)_{A_s}}{9.6\ \text{MeV}}\right) + \left(\dfrac{Z-\sqrt{1/\alpha_s}}{4\pi}\right)^2 \\ A_s \approx 2Z + \left(\dfrac{Z}{4\pi}\right)^2 \cong 2Z + kZ^2 \end{cases}$$

Note: With $(R_0/2) \cong 0.62$ fm, for $Z > 22$, it possible to fit the root mean square charge radii [7] with error less than 2%. See section-8.

With reference to stable mass number, approximately, it is possible to show that, for $(Z > 30)$,

$$\left.\begin{aligned}(B)_{A_s} &\approx \left[A_s - k\left(Z-\sqrt{1/\alpha_s}\right)^2\right]\left(\left(\frac{1}{\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_0}\right) \\ &\approx \left[A_s - k\left(Z-\sqrt{1/\alpha_s}\right)^2\right]\times 9.6\ \text{MeV}\end{aligned}\right\} \quad (10)$$

Case-2: Considering $Z = 30$ as a characteristic reference number and considering our previous publications [6,7], binding energy of light atomic nuclides can be approximately fitted by the following relation. For $(Z < 30)$,

$$\left.\begin{aligned}(B)_{A_s} &\cong \left(\frac{Z}{30}\right)^{\frac{1}{6}}\left(\frac{1}{\alpha_s}\right)\frac{2Ze^2}{4\pi\varepsilon_0 R_0} \\ &\cong \left(\frac{Z}{30}\right)^{\frac{1}{6}}\left(\frac{1}{\alpha_s}\right)\frac{Ze^2}{4\pi\varepsilon_0 (R_0/2)}\end{aligned}\right\} \quad (11)$$

Due to coulombic repulsion, in light atomic nuclides, strength of nuclear binding energy seems to decrease by a factor $\left(\frac{Z}{30}\right)^{\frac{1}{6}}$ and approaches $\left(\frac{Z}{30}\right)^{\frac{1}{6}}\left(\frac{1}{\alpha_s}\right)$. We are working in this direction. Numerically it is also possible to show that,

$$\left(\frac{Z}{30}\right)^{\frac{1}{6}}\left(\frac{1}{\alpha_s}\right) \approx \left\{\left(\frac{1+\alpha_s}{2\alpha_s}\right) + \ln(Z)\right\} \quad (12)$$

Based on this kind of relation, if $(R_0/2) \cong 0.61$ fm, relation (12) can be modified as,

$$\left.\begin{aligned}(B)_{A_s} &\approx \left\{\left(\frac{1+\alpha_s}{2\alpha_s}\right) + \ln(Z)\right\}\times\frac{Ze^2}{4\pi\varepsilon_0 (R_0/2)} \\ &\approx \left\{\left(\frac{1+\alpha_s}{2\alpha_s}\right) + \ln(Z)\right\}\times Z\times 2.36\ \text{MeV} \\ \text{and}\ A_s &\cong 2Z + (Z/4\pi)^2 \cong 2Z + kZ^2\end{aligned}\right\} \quad (13)$$

Proceeding further, by eliminating the bracket expression, for Z=1, binding energy of Deuteron can be shown to be 2.36 MeV. From this, it can be suggested that, in deuteron, there exists no strong interaction in-between proton and neutron.

Case-3: With reference to the semi empirical mass formula, approximately, below and above the stable mass number, nuclear binding energy can be expressed in the following form [7, 12, 13].

$$\left.\begin{aligned}&B_{(Z,A)} \cong \left(\frac{A-Z}{A_s-Z}\right)^p (B)_{A_s} \\ &\text{if}\ (A < A_s),\ p \cong (2/3);\ \text{if}\ (A > A_s),\ p \cong (1/2)\end{aligned}\right\} \quad (14)$$

7. Nuclear binding energy with respect to mass number and proton number

a) Proposed 8.9 MeV can also be understood with:

$$B_0 \cong \sqrt{\frac{1}{k}}(a_c) \cong \sqrt{k}\alpha_s\left(m_p c^2\right) \cong \sqrt{\alpha_s\left(m_p c^2\right)(a_c)} \quad (15)$$

b) With reference to strong coupling constant, stable mass number and chosen mass number close to the beta stability line,

$$B_A \cong \gamma\left\{A - \left[kN\left(\frac{kAA_s}{\sqrt{Z}} + 1\right)\right]\right\}\times(B_0 \cong 8.9\ \text{MeV}) \quad (16)$$

Here, for $Z < 25$, $\gamma \approx \alpha_s\left[\left(\frac{\alpha_s+1}{2\alpha_s}\right) + A^{\frac{1}{3}}\right]$

and for $Z \geq 25$, $\gamma \cong 1.0$

c) It is possible to split the above relation (16) into the following two possible forms.

For $Z < 25$,

$$B_A \approx \left[\left(\frac{\alpha_s+1}{2\alpha_s}\right)+\mathrm{A}^{\frac{1}{3}}\right]\left\{A-\left[kN\left(\frac{kAA_s}{\sqrt{Z}}+1\right)\right]\right\}\times 1.056 \text{ MeV} \quad (17)$$

For $Z \geq 25$,

$$B_A \cong \left(\frac{1}{\alpha_s}\right)\left\{A-\left[kN\left(\frac{kAA_s}{\sqrt{Z}}+1\right)\right]\right\}\times 1.056 \text{ MeV} \qquad (18)$$

d) Important points to be noted

i. At $Z=2$, the strength of the binding energy starts from $\left[\left(\frac{\alpha_s+1}{2\alpha_s}\right)+(4)^{\frac{1}{3}}\right]\cong 6.3$ and reaches the value of $\left[\left(\frac{\alpha_s+1}{2\alpha_s}\right)+(55)^{\frac{1}{3}}\right]\cong\frac{1}{\alpha_s}\cong 0.1186$ at $Z=25$.

ii. From $Z=25$ onwards, strength of binding energy seems to remain constant at $\frac{1}{\alpha_s}\cong 0.1186$.

iii. For Z=1 and A=2, by eliminating the bracket term $\left[\left(\frac{\alpha_s+1}{2\alpha_s}\right)+\mathrm{A}^{\frac{1}{3}}\right]$, binding energy of deuteron can be expressed as, $B_A \cong (2)\times 1.06 \text{ MeV} \cong 2.12 \text{ MeV}$.

8. To estimate the nuclear RMS charge radii

With the proposed characteristic nuclear charge radius of 0.62 fm, RMS charge radii [14,15] can be understood as:

$$R_{(Z,N)} \cong \left\{(Z)^{\frac{1}{3}}+(ZN)^{\frac{1}{6}}\right\}0.62 \text{ fm} \qquad (19)$$

This relation is free from arbitrary numbers and can be compared with the following relation available in current literature [13].

$$R_{(Z,N)} \cong \left\{1-0.349\left(\frac{N-Z}{N}\right)\right\}N^{\frac{1}{3}}\ 1.262 \text{ fm} \qquad (20)$$

9. Discussion

a) According to the relations (7), (8), (9), (10), (15) and (16) one can understand the physical significance of the new number $k \cong (1/4\pi)^2$.

b) Relations (14) and (16) can be recommended for further study.

c) We are working on $\left(\frac{Z}{30}\right)^{1/6}\left(\frac{1}{\alpha_s}\right)$, $\left\{\left(\frac{1+\alpha_s}{2\alpha_s}\right)+\ln(Z)\right\}$ and $\left[\left(\frac{\alpha_s+1}{2\alpha_s}\right)+\mathrm{A}^{\frac{1}{3}}\right]$ for finding a better relation for accuracy in estimating the binding energy of light atomic nuclides.

d) By fine tuning the values of $(\alpha_s \text{ and } R_0)$, better results can be achieved.

e) See table 1 for nuclear binding energy of stable and unstable isotopes of Z=40,50 and 60 estimated from relations (7), (9) and (14).

f) See table 2 for nuclear binding energy estimated from relations (7), (17) and (18).

g) See table 3 for nuclear binding energy of natural isotopes estimated from relations (7), (17) and (18).

h) See table 4 for nuclear RMS charge radii estimated from relations (19) and (20).

Table 1. *Comparison of isotopic nuclear binding energy of Z = 40,50 and 60 estimated from relations (7), (9) and (14)*

Proton number	Neutron number	Mass number	Estimated binding energy (MeV)	Thomas_Fermi binding energy [2]	%Error
40	40	80	675.6	669.27	-0.95
40	41	81	686.8	680.5	-0.93
40	42	82	698.0	692.31	-0.82
40	43	83	709.0	703.25	-0.82
40	44	84	720.0	717.24	-0.38
40	45	85	730.8	727.6	-0.44
40	46	86	741.6	740.82	-0.11
40	47	87	752.3	750.64	-0.22
40	48	88	763.0	763.17	0.03
40	49	89	773.5	772.27	-0.16
40	**50**	**90**	**784.0**	**784.06**	**0.01**
40	51	91	791.8	790.47	-0.17
40	52	92	799.5	799.34	-0.02
40	53	93	807.2	805.16	-0.25
40	54	94	814.8	813.27	-0.18
40	55	95	822.3	818.91	-0.41
40	56	96	829.7	827.03	-0.32
40	57	97	837.1	832.46	-0.56
40	58	98	844.4	840.46	-0.47
40	59	99	851.6	845.62	-0.71
40	60	100	858.8	853.02	-0.68
40	61	101	866.0	857.67	-0.97
40	62	102	873.0	864.42	-1.00
50	50	100	814.4	825.98	1.40
50	51	101	825.2	837.19	1.43
50	52	102	836.0	850.66	1.73
50	53	103	846.7	860.68	1.63
50	54	104	857.3	873.13	1.81
50	55	105	867.8	882.72	1.69
50	56	106	878.3	894.58	1.82
50	57	107	888.8	903.51	1.63
50	58	108	899.1	914.94	1.73
50	59	109	909.4	923.53	1.53
50	60	110	919.7	934.7	1.61
50	61	111	929.9	942.89	1.38
50	62	112	940.0	953.5	1.42
50	63	113	950.1	961.1	1.15
50	64	114	960.1	971.41	1.16
50	65	115	970.1	978.73	0.88
50	**66**	**116**	**980.0**	**988.47**	**0.86**
50	67	117	987.4	995.44	0.81

50	68	118	994.7	1004.74	1.00
50	69	119	1002.0	1011.26	0.91
50	70	120	1009.3	1020.32	1.08
50	71	121	1016.4	1026.76	1.00
50	72	122	1023.6	1035.51	1.15
50	73	123	1030.7	1041.54	1.04
50	74	124	1037.7	1050.08	1.18
50	75	125	1044.7	1055.84	1.06
50	76	126	1051.6	1064.06	1.17
50	77	127	1058.5	1069.61	1.04
50	78	128	1065.4	1077.54	1.13
50	79	129	1072.2	1082.82	0.98
50	80	130	1078.9	1090.52	1.06
50	81	131	1085.7	1095.61	0.91
50	82	132	1092.3	1102.55	0.93
50	83	133	1099.0	1105.23	0.56
50	84	134	1105.6	1109.54	0.36
50	85	135	1112.2	1111.44	-0.06
50	86	136	1118.7	1115.23	-0.31
60	60	120	947.2	946.38	-0.09
60	61	121	957.7	959.03	0.14
60	62	122	968.2	973.61	0.56
60	63	123	978.5	985.36	0.69
60	64	124	988.9	999.18	1.03
60	65	125	999.2	1010.33	1.11
60	66	126	1009.4	1023.59	1.39
60	67	127	1019.5	1034.26	1.42
60	68	128	1029.7	1046.96	1.65
60	69	129	1039.7	1057.1	1.64
60	70	130	1049.8	1069.13	1.81
60	71	131	1059.7	1078.61	1.75
60	72	132	1069.7	1090	1.87
60	73	133	1079.5	1098.78	1.75
60	74	134	1089.4	1109.79	1.84
60	75	135	1099.2	1118.45	1.72
60	76	136	1108.9	1129.1	1.79
60	77	137	1118.6	1137.46	1.66
60	78	138	1128.3	1147.92	1.71
60	79	139	1137.9	1156.26	1.59
60	80	140	1147.5	1167.12	1.68
60	81	141	1157.0	1175.37	1.56
60	82	142	1166.5	1185.62	1.61
60	**83**	**143**	**1176.0**	**1191.53**	**1.30**
60	84	144	1183.1	1199.07	1.33
60	85	145	1190.1	1204.16	1.17
60	86	146	1197.1	1211.9	1.22

60	87	147	1204.0	1217.65	1.12
60	88	148	1210.9	1225.22	1.17
60	89	149	1217.8	1230.73	1.05
60	90	150	1224.6	1238.33	1.11
60	91	151	1231.4	1243.73	0.99
60	92	152	1238.1	1250.81	1.01
60	93	153	1244.8	1255.77	0.87
60	94	154	1251.5	1262.35	0.86
60	95	155	1258.1	1266.85	0.69
60	96	156	1264.7	1272.97	0.65
60	97	157	1271.3	1277.14	0.46
60	98	158	1277.9	1282.89	0.39
60	99	159	1284.4	1286.74	0.19

Table 2. *Comparison of estimated nuclear binding energy and Thomas_Fermi binding energy [Relations (7), (17) and (18)]*

Proton number	**Estimated Mass number**	**Neutron number**	**Strength of binding energy**	**Estimated binding energy (MeV)**	**Thomas_Fermi binding energy [2] (MeV)**	**Error (MeV)**
2	4	2	6.3033	26.5	29.80	3.3
3	6	3	6.5330	41.2	32.99	-8.3
4	8	4	6.7159	56.5	58.08	1.6
5	10	5	6.8703	72.3	65.93	-6.3
6	12	6	7.0053	88.4	93.41	5.0
7	14	7	7.1260	104.9	105.61	0.7
8	16	8	7.2357	121.6	128.57	6.9
9	19	10	7.3843	147.3	149.37	2.1
10	21	11	7.4748	164.7	167.98	3.3
11	23	12	7.5597	182.4	187.09	4.7
12	25	13	7.6399	200.3	206.14	5.9
13	27	14	7.7159	218.3	225.57	7.2
14	29	15	7.7882	236.6	245.07	8.5
15	31	16	7.8572	255.1	262.91	7.9
16	34	18	7.9555	282.9	291.84	8.9
17	36	19	8.0178	301.8	306.79	5.0
18	38	20	8.0778	320.7	327.93	7.2
19	40	21	8.1358	339.9	342.14	2.3
20	43	23	8.2192	368.6	371.58	2.9
21	45	24	8.2727	388.1	389.31	1.2
22	47	25	8.3247	407.6	407.82	0.2
23	49	26	8.3752	427.3	426.34	-1.0
24	52	28	8.4484	456.8	457.31	0.5
25	54	29	8.4317	473.1	472.33	-0.8
26	56	30	8.4317	490.3	491.73	1.4

27	59	32	8.4317	515.9	516.46	0.6
28	61	33	8.4317	533.0	534.10	1.1
29	63	34	8.4317	550.1	549.96	-0.1
30	66	36	8.4317	575.4	577.16	1.8
31	68	37	8.4317	592.4	590.61	-1.8
32	70	38	8.4317	609.4	609.81	0.4
33	73	40	8.4317	634.5	634.34	-0.1
34	75	41	8.4317	651.4	651.02	-0.3
35	78	43	8.4317	676.3	676.11	-0.2
36	80	44	8.4317	693.1	694.89	1.8
37	83	46	8.4317	717.8	720.46	2.6
38	85	47	8.4317	734.5	737.85	3.3
39	88	49	8.4317	759.1	763.88	4.8
40	90	50	8.4317	775.7	784.06	8.4
41	93	52	8.4317	800.0	805.49	5.4
42	95	53	8.4317	816.5	820.94	4.4
43	98	55	8.4317	840.7	843.40	2.7
44	100	56	8.4317	857.1	861.42	4.3
45	103	58	8.4317	881.1	883.90	2.8
46	105	59	8.4317	897.3	899.50	2.2
47	108	61	8.4317	921.1	922.20	1.1
48	111	63	8.4317	944.8	947.19	2.4
49	113	64	8.4317	960.8	962.83	2.0
50	116	66	8.4317	984.3	988.47	4.2
51	118	67	8.4317	1000.2	1000.48	0.3
52	121	69	8.4317	1023.4	1024.43	1.0
53	124	71	8.4317	1046.5	1046.32	-0.2
54	126	72	8.4317	1062.3	1063.20	0.9
55	129	74	8.4317	1085.1	1085.08	-0.1
56	132	76	8.4317	1107.8	1109.24	1.4
57	135	78	8.4317	1130.4	1131.00	0.6
58	137	79	8.4317	1145.8	1145.70	-0.1
59	140	81	8.4317	1168.2	1168.67	0.5
60	143	83	8.4317	1190.3	1191.53	1.2
61	146	85	8.4317	1212.3	1209.52	-2.8
62	148	86	8.4317	1227.5	1224.89	-2.6
63	151	88	8.4317	1249.2	1244.78	-4.5
64	154	90	8.4317	1270.8	1267.23	-3.6
65	157	92	8.4317	1292.3	1287.38	-4.9
66	160	94	8.4317	1313.5	1309.48	-4.1
67	162	95	8.4317	1328.3	1321.18	-7.1
68	165	97	8.4317	1349.3	1343.08	-6.2
69	168	99	8.4317	1370.1	1363.31	-6.8
70	171	101	8.4317	1390.8	1384.86	-6.0
71	174	103	8.4317	1411.3	1404.44	-6.9
72	177	105	8.4317	1431.6	1425.49	-6.1
73	180	107	8.4317	1451.8	1444.94	-6.8

74	183	109	8.4317	1471.7	1465.60	-6.1
75	186	111	8.4317	1491.5	1484.63	-6.9
76	189	113	8.4317	1511.1	1504.86	-6.3
77	192	115	8.4317	1530.5	1523.81	-6.7
78	195	117	8.4317	1549.8	1544.71	-5.1
79	198	119	8.4317	1568.8	1564.94	-3.9
80	201	121	8.4317	1587.7	1586.70	-1.0
81	204	123	8.4317	1606.3	1606.87	0.5
82	207	125	8.4317	1624.8	1628.12	3.3
83	210	127	8.4317	1643.1	1643.94	0.8
84	213	129	8.4317	1661.2	1659.72	-1.5
85	216	131	8.4317	1679.1	1673.42	-5.7
86	219	133	8.4317	1696.8	1690.59	-6.2
87	222	135	8.4317	1714.3	1706.49	-7.8
88	225	137	8.4317	1731.6	1724.18	-7.4
89	228	139	8.4317	1748.7	1740.67	-8.1
90	231	141	8.4317	1765.6	1759.14	-6.5
91	234	143	8.4317	1782.3	1776.08	-6.2
92	238	146	8.4317	1803.0	1800.77	-2.2
93	241	148	8.4317	1819.2	1817.31	-1.9
94	244	150	8.4317	1835.2	1835.45	0.3
95	247	152	8.4317	1850.9	1851.73	0.8
96	250	154	8.4317	1866.5	1868.97	2.5
97	254	157	8.4317	1885.5	1888.79	3.3
98	257	159	8.4317	1900.5	1906.19	5.7
99	260	161	8.4317	1915.3	1922.20	6.9
100	263	163	8.4317	1929.9	1939.52	9.6

Table 3. *Comparison of estimated nuclear binding energy and Thomas_Fermi binding energy for natural isotopes [Relations (7), (17) and (18)]*

Proton number	Neutron number	Mass number	Binding strength	Estimated binding energy (MeV)	Thomas_Fermi binding energy [2] (MeV)	Error (MeV)
8	8	16	7.24	121.6	127.619	6.0
8	9	17	7.29	130.1	131.763	1.7
8	10	18	7.34	138.6	139.807	1.2
12	12	24	7.60	191.3	198.257	6.9
12	13	25	7.64	200.3	205.588	5.3
12	14	26	7.68	209.2	216.681	7.4
12	15	27	7.72	218.3	224.952	6.7
16	16	32	7.89	264.3	271.781	7.4
16	17	33	7.92	273.6	280.422	6.8
16	18	34	7.96	282.9	291.839	8.9
16	20	36	8.02	301.7	308.714	7.1
20	20	40	8.14	339.9	342.052	2.1
20	22	42	8.19	359.0	361.896	2.9
20	23	43	8.22	368.6	369.829	1.2
20	24	44	8.25	378.3	380.96	2.7
20	26	46	8.30	397.6	398.769	1.1
24	26	50	8.40	437.1	435.049	-2.1
24	28	52	8.45	456.8	456.349	-0.5
24	29	53	8.47	466.7	464.289	-2.4
24	30	54	8.50	476.6	474.008	-2.6
26	28	54	8.43	473.3	471.763	-1.5
26	30	56	8.43	490.3	492.258	1.9
26	31	57	8.43	498.8	499.905	1.1
26	32	58	8.43	507.3	509.949	2.6
36	42	78	8.43	676.5	675.578	-1.0
36	44	80	8.43	693.1	695.434	2.4
36	46	82	8.43	709.6	714.274	4.7
36	47	83	8.43	717.8	721.737	3.9
36	48	84	8.43	726.0	732.258	6.2
46	56	102	8.43	873.5	875.212	1.8
46	58	104	8.43	889.4	892.82	3.4
46	59	105	8.43	897.3	899.914	2.6
46	60	106	8.43	905.3	909.474	4.2
46	62	108	8.43	921.1	925.239	4.1
46	64	110	8.43	936.9	940.207	3.3
56	74	130	8.43	1092.7	1092.722	0.0
56	76	132	8.43	1107.8	1110.038	2.2
56	78	134	8.43	1122.9	1126.696	3.8
56	79	135	8.43	1130.4	1133.668	3.3
56	80	136	8.43	1137.9	1142.775	4.9
56	81	137	8.43	1145.4	1149.681	4.3
56	82	138	8.43	1152.8	1158.293	5.5

66	90	156	8.43	1285.2	1278.021	-7.2
66	92	158	8.43	1299.4	1294.046	-5.3
66	94	160	8.43	1313.5	1309.455	-4.1
66	95	161	8.43	1320.6	1315.909	-4.7
66	96	162	8.43	1327.6	1324.106	-3.5
66	97	163	8.43	1334.7	1330.377	-4.3
66	98	164	8.43	1341.7	1338.035	-3.6
76	108	184	8.43	1478.4	1469.921	-8.5
76	110	186	8.43	1491.5	1484.807	-6.7
76	111	187	8.43	1498.1	1491.097	-7.0
76	112	188	8.43	1504.6	1499.087	-5.5
76	113	189	8.43	1511.1	1505.007	-6.1
76	114	190	8.43	1517.6	1512.799	-4.8
76	116	192	8.43	1530.6	1526.116	-4.5
82	122	204	8.43	1606.3	1607.507	1.2
82	124	206	8.43	1618.7	1622.325	3.6
82	125	207	8.43	1624.8	1629.063	4.2
82	126	208	8.43	1631.0	1636.431	5.5
92	142	234	8.43	1781.1	1778.567	-2.5
92	143	235	8.43	1786.6	1783.864	-2.7
92	146	238	8.43	1803.0	1801.69	-1.3

Table 4. *To fit the RMS charge radii of* $Z > 22$ *estimated from relations (19) and (20)*

Proton number	Neutron number	Mass number	Estimated RMS charge radius Relation (19) (fm)	RMS charge radius [14] Relation (20) (fm)	Reference RMS charge radius [15] (fm)
23	28	51	3.585	3.5933	3.6002
24	28	52	3.623	3.6411	3.6452
25	30	55	3.682	3.6932	3.7057
26	30	56	3.718	3.7389	3.7377
27	32	59	3.773	3.7881	3.7875
28	33	61	3.818	3.8339	3.8399
29	34	63	3.861	3.8786	3.8823
30	36	66	3.912	3.9246	3.9491
31	38	69	3.963	3.9700	3.9973
32	38	70	3.994	4.0090	4.0414
33	42	75	4.059	4.0587	4.0968
34	42	76	4.089	4.0951	4.1395
35	44	79	4.135	4.1373	4.1629
36	44	80	4.164	4.1726	4.1970
37	46	83	4.208	4.2130	4.2058
38	48	86	4.252	4.2529	4.2307
39	50	89	4.294	4.2923	4.2430
40	50	90	4.321	4.3247	4.2694
41	52	93	4.362	4.3627	4.3240
42	53	95	4.395	4.3971	4.3628
44	56	100	4.467	4.4672	4.4531
45	58	103	4.506	4.5029	4.4945
46	59	105	4.537	4.5352	4.5150
47	60	107	4.568	4.5670	4.5454
48	63	111	4.611	4.6043	4.5845
49	64	113	4.641	4.6351	4.6010
50	66	116	4.676	4.6686	4.6250
52	68	120	4.734	4.7281	4.7038
53	74	127	4.791	4.7736	4.7500
54	72	126	4.802	4.7921	4.7722
55	74	129	4.835	4.8235	4.7981
56	76	132	4.868	4.8547	4.8303
57	78	135	4.900	4.8854	4.8488
58	78	136	4.921	4.9096	4.8739
59	82	141	4.963	4.9460	4.8919
60	83	143	4.989	4.9725	4.9254
62	86	148	5.045	5.0279	5.0042
63	88	151	5.075	5.0568	5.0522
64	90	154	5.105	5.0853	5.1223
65	92	157	5.134	5.1136	5.0489
66	94	160	5.163	5.1416	5.1951
67	96	163	5.192	5.1693	5.1907

68	96	164	5.211	5.1903	5.2389
69	100	169	5.248	5.2239	5.2236
70	101	171	5.271	5.2476	5.2906
71	104	175	5.303	5.2776	5.3700
72	105	177	5.326	5.3007	5.3309
73	108	181	5.357	5.3302	5.3507
74	109	183	5.379	5.3528	5.3611
75	110	185	5.402	5.3753	5.3596
76	113	189	5.432	5.4040	5.4016
77	114	191	5.454	5.4261	5.3968
78	117	195	5.483	5.4544	5.4270
79	118	197	5.505	5.4759	5.4371
80	121	201	5.534	5.5038	5.4581
81	122	203	5.555	5.5250	5.4666
82	125	207	5.583	5.5524	5.4943
83	126	209	5.604	5.5732	5.5211
84	132	216	5.643	5.6102	5.6359
86	132	218	5.676	5.6442	5.6540
87	134	221	5.700	5.6674	5.6790
88	137	225	5.727	5.6937	5.7150
90	142	232	5.776	5.7425	5.7848
92	146	238	5.822	5.7875	5.8571
94	146	240	5.853	5.8192	5.8701
95	148	243	5.875	5.8411	5.9048

10. Conclusion

Semi empirical mass formula and Fermi gas model, both, are lagging in implementing the strong coupling constant in nuclear structure. In this context, understanding and estimating nuclear binding energy with 'strong interaction' concepts seems to be quite interesting and needs a serious review at basic level. So far we have developed 3 to 4 versions with many possible relations. We believe that, results obtained from above relations are simple to understand and seem to be more physical and relatively closer to the experimental data. We are working on deriving the above relations and confident to say that, by following the proposed kind of semi empirical relations, one can certainly implement the strong coupling constant in nuclear binding energy scheme successfully. With further research, current nuclear models and strong interaction concepts pertaining to high energy physics can be studied in a unified manner and a realistic nuclear model can be developed.

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Conflict of Interest

The authors have no conflict of interest.

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