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## A study on estimation of electric quadrupole transition probability in nuclei

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### ABSTRACT

The reduced electric quadrupole transition probability ( $B(E2)\uparrow$ ) between  $0^+$  ground state and  $2^+$  state in nuclei is an important quantity because it represents basic nuclear information on energies of low-lying levels in the nuclei. It provides knowledge about deformation of nuclei. In this study, the  $B(E2)\uparrow$  values of some even-even nuclei in  $110 \leq A \leq 190$  region have been estimated by using artificial neural network (ANN) method which is a non-linear approximator. The present study shows that ANN is found to be useful in order to predict  $B(E2)\uparrow$  values of even-even nuclei in this region.

**Keywords:** Electric quadrupole transition probability, atomic structure, artificial neural network

### 1. Introduction

One of the fundamental properties of the nuclei is their shapes. Nuclei with magic numbers of neutron and proton have a closed shell. Nuclei with neutron (N) or proton (Z) numbers far from a magic number generally have deformed shape. The simplest deformations are called quadrupole deformations where the nuclei can either take an oblate or a prolate shape. The reduced electric quadrupole transition probability ( $B(E2)\uparrow$ ) includes nuclear information about energy of low-lying levels of nuclei. The first excited states of the even-even nuclei are  $2^+$ . So, the transition from this state to the  $0^+$  ground state is important. It is highly related to nuclear quadrupole deformation parameter ( $\beta$ ), mean lifetime ( $\tau$ ) and electric quadrupole moment ( $Q_0$ ) by

$$\beta = \left( \frac{4\pi}{3ZR_0^2} \right) [B(E2)\uparrow \frac{1}{e^2}]^{1/2} \quad (1)$$

$$\tau = \frac{40.81 \times 10^{13} E^{-5}}{[B(E2)\uparrow / e^2 b^2] (1 + \alpha)} \quad (2)$$

$$Q_0 = \left[ \frac{16\pi}{5} \times B(E2)\uparrow \frac{1}{e^2} \right]^{1/2} \quad (3)$$

where E is the energy of the first excited  $2^+$  state,  $\alpha$  is the total conversion coefficient, Z is the proton number of nuclei and  $R_0 = 1.2 \times 10^{-13} A^{1/3}$  cm. The deformations of nuclei are important for understanding their shapes (prolate, oblate etc.) and structures. The lifetimes of the levels are useful for determining the energy levels in nuclei. So, there is much attention in  $B(E2)\uparrow$  value.

The reduced electric quadrupole transition probability is measured by inelastic electron scattering, muonic x-ray measurement, Mössbauer spectroscopy, Coulomb excitation, lifetime measurement or resonance fluorescence [1]. There are also several theoretical model for prediction of the  $B(E2)\uparrow$  values based on single-shell asymptotic Nilsson model [2], finite-range droplet model [3], Woods-Saxon model [4], relativistic mean-field model [5], extended Thomas-Fermi Strutinsky-Integral method [6], Hartree-Fock+BCS method [7] and dynamical microscopic model [8]

Recently, artificial neural network (ANN) has been used in many fields in nuclear physics such as developing nuclear mass systematic [9], identification of impact parameter in heavy-ion

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collisions [10-12], estimating beta decay half-lives [13], neutron-gamma separation in order to obtain clear gamma-ray spectra [14], prediction of peak-to-background ratio in gamma-ray spectroscopy [15] and obtaining nuclear charge radii [16]. In this study, feed-forward ANN has been used to estimate  $B(E2)\uparrow$  values for some even-even nuclei between  $110 \leq A \leq 190$ . The adopted values of the  $B(E2)\uparrow$  have been obtained from Ref. [1]. In the work performed by Raman et al. [1], due to the fact that there are several  $B(E2)\uparrow$  values for the nuclei, the more reliable weighting values has been used. The main aim of the present study is to show success of the ANN in describing of the  $B(E2)\uparrow$  values of nuclei by using known data.

## 2. Artificial Neural Network (ANN)

A mathematical model that mimics the brain functionality is called as artificial neural network (ANN) [17]. This method is a perfect tool which does not need any relationship between the data. There are two main class of data about the problem considered, one is input and the other is desired (output). ANN is composed of different main layers. These are input, output and hidden layers. Input and output layers include input and output data, respectively. Each layer has one or more processing units called neurons which are connected to each other in the next layers by adaptive synaptic weights. By transmitting the data between neurons in different layers, the communication is performed. The aim is the determination of the weight values. The input neurons receive the data from outside. The most used activation function for the hidden neurons is a sigmoid-like function like tangent hyperbolic,  $\tanh = (e^x - e^{-x}) / (e^x + e^{-x})$ . The output neurons in the last layer give the result. The input and output neuron numbers depend on the variety of the input and output data, respectively. Besides, the number of neurons in the hidden layer (h) can differ. Generally, as the number of h increases, the predictions get better.

In our calculation we have used ANN with four layers in order to estimate reduced transition probability in nuclei. The input layer consist of two neurons. One is for proton number (Z) and the other is for neutron number (N) of the nuclei. After several trials, the hidden layer and the neuron numbers have been chosen as 2 and 4, respectively. The output layer with one neuron corresponds to adopted values of the reduced transition probability ( $B(E2)\uparrow$ ). The architecture of the ANN has been 2 - 4 - 4 - 1 (Fig. 1) and the total number of adjustable weights has been 28 according to the formula given as

$$w = p \times h_1 + h_1 \times h_2 + \dots + h_i \times r \quad (4)$$

where w is the number of total weights,  $h_1$ ,  $h_2$  and  $h_i$  are hidden neuron numbers in first, second and  $i^{\text{th}}$  hidden layers, respectively, p and r are the numbers of the input and output layers, respectively.

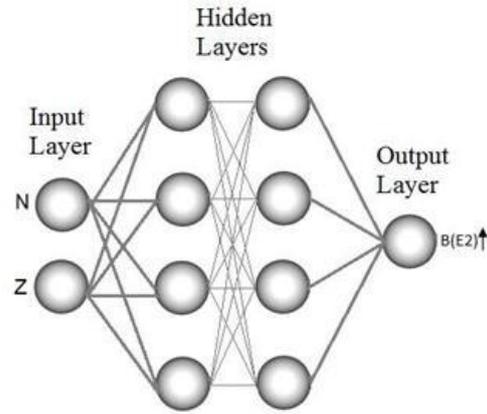


Fig. 1. ANN architecture (2-4-4-1) used in this work.

The ANN method has been composed of two main steps: training and test. In the supervised training step, the ANN has been constructed by using known input and output data. The weights values of each connections between the neurons have been adjusted for this construction. Until a predetermined acceptable error level, the construction process continues. In this study, a back-propagation algorithm with Levenberg-Marquardt [18,19] has been used for adjusting the connections in order to obtain agreement between neural network output and desired output. The error function which evaluates the difference between different outputs is mean square error (MSE) given as

$$MSE = \frac{[\sum_{k=1}^r \sum_{i=1}^N (y_{ki} - f_{ki})^2]}{N} \quad (5)$$

where  $y_{ki}$  and  $f_{ki}$  are desired and neural network outputs, respectively, N is the number of training and test samples, whichever applies. After the first step in ANN process, the second step (test) is started. When the unknown data are provided as inputs which are not used in the training step, it is expected to obtain ANN outputs. If the difference between the different outputs is acceptably small, it has been concluded that ANN has generalized the data. Therefore, this constructed ANN can be used safely for all the same group of data.

## 3. Result and Discussions

In this work, the adopted values of  $B(E2)\uparrow$  [1] have been used in ANN. The unit of the  $B(E2)\uparrow$  value has been given by  $e^2b^2$ . The nuclei between  $A=110$  and  $A=190$  have been considered in the process due to the high deformations in this region. The adopted values of the nuclei whose atomic numbers are from

$Z = 52$  to  $Z = 74$  in the given mass region have been used for the training procedure. The trained ANN has been tested first on the training data in order to see the learning capability of the network. As can be clearly seen in the Fig. 2 that the deviations from adopted values are between 0.1 and -0.1 and mostly concentrated near zero. The MSE value belonging to the training data is  $2.3 \times 10^{-2}$ . Although the large deformation has been seen in the figures, these ANN predictions remain within the error limits of the adopted levels. Therefore, it can safely be concluded that the ANN construction for the prediction is successfully completed. Also seen in the figure that, the  $B(E2)\uparrow$  values are minimum for near the closed shell nuclei and maximum in the middle of a shell.

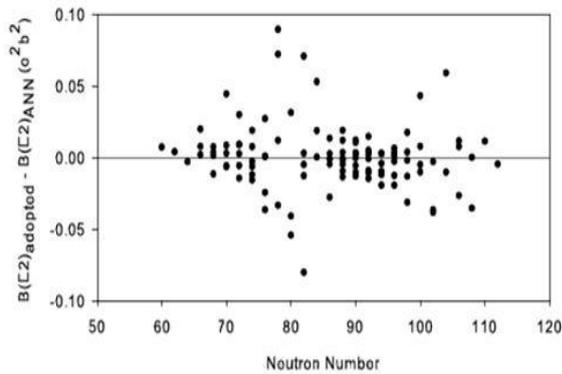


Fig. 2. The differences between adopted values and ANN outputs for  $B(E2)\uparrow$  values for training data.

After the construction of the ANN in training process, the network has been tested over the data which are new for the network. These test nuclei have been Ba, Ce, Nd, Sm, Gd and Dy. According to the results, the MSE values have been lied between  $0.8 \times 10^{-3}$  and  $2.2 \times 10^{-3}$ . These small values of the MSE indicate that the test set ANN has consistently generalized the training set fittings. The estimations belonging to these nuclei is shown in Fig. 3 in which the adopted values and ANN estimations for  $B(E2)\uparrow$  are given in the same graph for comparison. As can be clearly seen in the Fig. 3 the estimations are consistent with the adopted values. There is no large difference between the adopted and ANN values in the abrupt decreases in the neutron magic number 82. Moreover, it has been more clear in the Nd and Sm examples that ANN has found the neutron magic number 82.

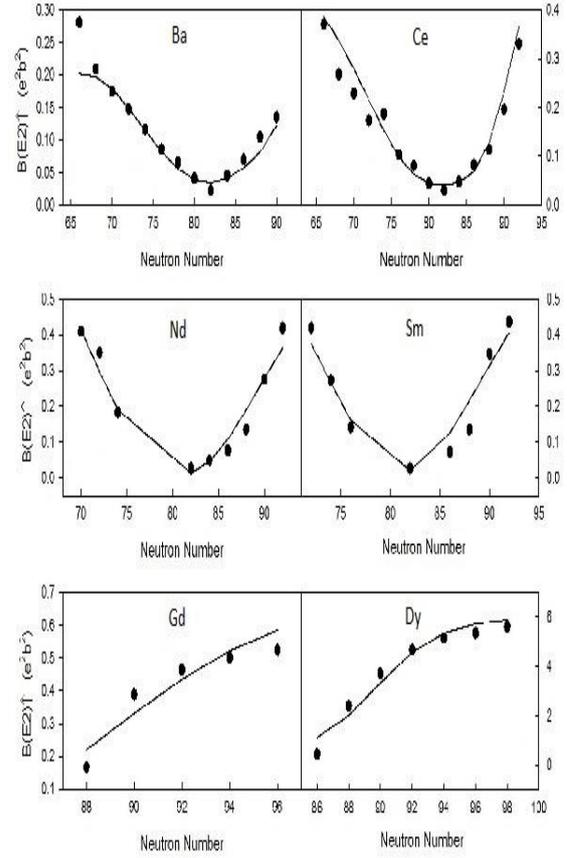


Fig. 3. Adopted values (circle) and ANN estimations (line) for  $B(E2)\uparrow$  of the Ba, Ce, Nd, Sm, Gd and Dy isotopes.

#### 4. Conclusions

In order to estimate reduced electric transition probability ( $B(E2)\uparrow$ ) for some even - even nuclei in the region  $A = 110 - 190$ , artificial neural network (ANN) method has been employed. The inputs have been adopted values of  $B(E2)$  which were produced and compiled before by Raman et al. [1], from different experiments. It has been seen in the present work that the prediction power of the ANN on Ba, Ce, Nd, Sm, Gd and Dy nuclei is high. The results from the method is found to be consistent with the adopted values of  $B(E2)\uparrow$ . The maximum deviations is seen in the nuclear magic numbers.

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