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Analytical Formulae to Calculate the Total Efficiency of an Arbitrarily Positioned Point Source by an Elliptical Cylindrical Detector

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ABSTRACT

In this paper, a direct analytical method is presented for calculating the absolute efficiency of an elliptical cylindrical detector in the case of an arbitrarily positioned (above the major axis a) point source. The absolute efficiency is required to determine the activity of an unknown radioactive source, taking into account the attenuation of the gamma-ray photons. The validity of the derived analytical expressions was successfully confirmed by the comparison with some published data.

Keywords: Analytical formula, Solid angle, Elliptical cylindrical detector, Absolute efficiency

1. Introduction

One of the most important parameters in the calculation of the gamma activity of environmental radioactive sources with respect to emitted gamma energy is the detection efficiency which is usually determined using calibrated standard sources [1]. Several authors [2-8] have been treated the issue concerning with the absolute efficiencies determination and have given some useful solutions to this problem. Recently, Selim and Abbas [9-15] calculated the total and full-energy peak efficiencies for any source-detector configuration using spherical coordinates system.

In the present work, a new and a simple theoretical approach is introduced for analytical formula to calculate absolute efficiency of an elliptical cylindrical detector in the case of an arbitrarily positioned (above the major axis a) point source. The present method combines the calculation of the average path length covered by photon inside the detector active volume and the solid angle. Several different cases were determined, depending on the particle range, radius of the detector and the position of the source with respect to the detector. In the present work, these cases were analyzed separately and different expressions for calculating the hit probability were obtained for each of them at an arbitrarily positioned (above the major axis a) radiating point source.

2. Mathematical viewpoint

The total efficiency of a gamma-ray detector, ε_{point}, using an arbitrarily positioned isotropic radiating point source is defined as

$$\varepsilon_{point} = \varepsilon_g \times \varepsilon_i \tag{1}$$

where ε_g is the geometric efficiency which is represented by equation

$$\varepsilon_g = \frac{\Omega}{4\pi} \tag{2}$$

and Ω is the solid angle subtended by the detector at an arbitrarily positioned radiating point source represented by equation

$$\Omega = \int_\theta \int_\phi \sin\theta d\phi d\theta \tag{3}$$

where ε_i is the intrinsic efficiency which is represented by equation

$$\varepsilon_i = \left(1 - e^{-\mu \bar{d}}\right) \tag{4}$$

where μ is the attenuation coefficient and $\bar{d}$ is the average path length traveled by a photon through the detector and is given by

$$\bar{d} = \frac{\int_\Omega d(\theta,\phi) d\Omega}{\int_\Omega d\Omega} = \frac{\int_\theta \int_\phi d(\theta,\phi) \sin\theta d\phi d\theta}{\Omega} \tag{5}$$

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where θ and ϕ are the polar and the azimuthal angles, respectively. $d(\theta,\phi)$ is the possible path length traveled by the photon within the detector active. For each photon emitted from the point source, the probability of striking the point where the photon actually enters the detector active volume must be known to calculate $\bar{d}$ and consequently the detection efficiency. The factor determining the photon attenuation by the source container and the detector and cap materials, f_{att}, is expressed as

$$f_{att} = e^{-\Sigma_i \mu_i \delta_i} \tag{6}$$

where μ_i is the attenuation coefficient of the ith absorber for a gamma-ray photon with energy E_γ and δ_i is the gamma-ray photon path length through the ith absorber.

The work described below involves the use of straightforward analytical formulae for the computation of the total efficiency subtended by an elliptical cylindrical detector at an arbitrarily positioned radiating point source.

2.1. The case of an isotropic radiating point source S (0, 0, h)

Consider an elliptical cylindrical *(a, b, L)* detector and an arbitrarily positioned isotropic point source located at a distance *h* from the center of the detector top surface, as shown in **Fig. 1.** The efficiency of the detector with respect to an arbitrarily positioned radiating point source is given by as follows

$$\varepsilon = \frac{1}{4\pi}\int_0^{2\pi}\left[\int_0^{\theta_1(\phi)} (1 - e^{-\mu.d_1(\phi)}) sin\theta \, d\theta + \int_{\theta_1(\phi)}^{\theta_2(\phi)} (1 - e^{-\mu.d_2(\phi)}) sin\theta d\theta\right] d\phi \tag{7}$$

where $d_1(\theta)$ is the photon path length traveled through the detector active volume, it enters from the upper face and emerge from its base, which is represented by:

$$d_1(\theta) = \frac{L}{cos\theta_1(\phi)} \tag{8}$$

while, $d_2(\theta)$ is the photon path length traveled through the detector active volume, it enters from the upper face and emerge from its side, which is represented by:

$$d_2(\theta) = \frac{r_1(\phi) - h.tan\theta_2(\phi)}{sin\theta_2(\phi)} \tag{9}$$

where the polars $\theta_1(\phi)$ and $\theta_2(\phi)$ and the azimuthal ϕ angles are given by respectively:

$$\theta_1(\phi) = \tan^{-1}\left(\frac{r_1(\phi)}{h + L}\right) \tag{10}$$

$$\theta_2(\phi) = \tan^{-1}\left(\frac{r_1(\phi)}{h}\right) \tag{11}$$

$$0 \le \phi \le 2\pi \tag{12}$$

where, *h* is the height from top surface of an elliptical cylindrical detector to an arbitrarily positioned radiating point source *S (0, 0, h)* and $r_1(\phi)$ is the distance on the polar form relative to center of an ellipse, which describes a general distance from the center to the circumference of an ellipse by rotating the major axis angle (ϕ) and with semi-diameters of major and minor axis *a* and

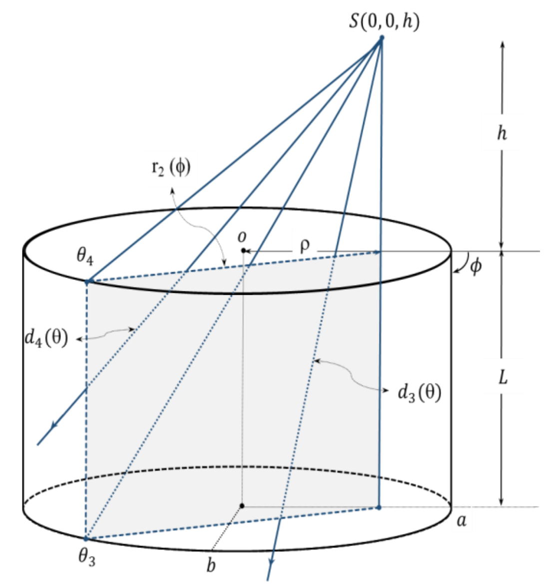

Fig. 1. Schematic view of an arbitrarily positioned radiating point source S (p, 0, h) located above the major axis of an elliptical cylindrical detector at a radial distance p <a and at a height h.

b respectively of an ellipse. The form of $r_1(\phi)$ is given by:

$$r_1(\phi) = \frac{a.b}{\sqrt{(a \cdot \sin\phi)^2 + (b \cdot \cos\phi)^2}} \tag{13}$$

The geometrical notations of *a, b* and *h* are as shown in Fig. 1. By combining Eqs. (7-13), results in direct mathematical expression for the total efficiency subtended by an elliptical cylindrical detector, from an isotropic radiating central point source *S (0, 0, h)*.

2.2. The case of an isotropic radiating point source *S (p, 0, h)*:

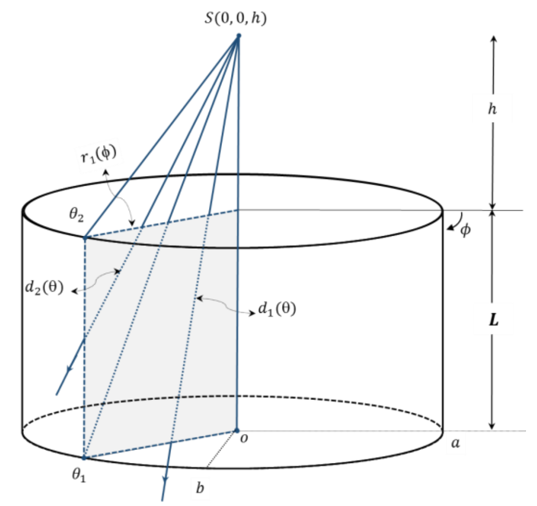

Fig. 2. Schematic view of an arbitrarily positioned radiating point source S(0,0,h) located above the center of an elliptical cylindrical detector at a height h.

Case I: $p = \rho < a$

Consider an elliptical cylindrical *(a, b, L)* detector and an arbitrarily positioned isotropic point source located at a distance *h* from the center of the detector top surface, as shown in Fig. 2.

The efficiency of the detector with respect to a point source is given as follows

$$\varepsilon = \frac{1}{4\pi}\int_0^{2\pi}\left[\int_0^{\theta_3(\phi)} (1-e^{-\mu.d_3(\phi)})\sin\theta\, d\theta + \int_{\theta_3(\phi)}^{\theta_4(\phi)} (1-e^{-\mu.d_4(\phi)})\sin\theta\, d\theta\right] d\phi \quad (14)$$

where $d_3(\theta)$ is the photon path length traveled through the detector active volume, it enters from the upper face and emerge from its base, which is represented by:

$$d_3(\theta) = \frac{L}{cos\theta_3(\phi)} \quad (15)$$

while $d_4(\theta)$ is the photon path length traveled through the detector active volume, it enters from the upper face and emerge from its side, which is represented by:

$$d_4(\theta) = \frac{r_2(\phi) - h.\,tan\,\theta_4(\phi)}{sin\theta_4(\phi)} \quad (16)$$

where the polars $\theta_3(\phi)$ and $\theta_4(\phi)$ and the azimuthal ϕ angles are given by respectivly:

$$\theta_3(\phi) = \tan^{-1}\left(\frac{r_2(\phi)}{h+L}\right) \quad (17)$$

$$\theta_4(\phi) = \tan^{-1}\left(\frac{r_2(\phi)}{h}\right) \quad (18)$$

$$0 \le \phi \le 2\pi \quad (19)$$

where *h* is the height from top surface of an elliptical cylindrical detector to an arbitrarily positioned radiating point source *S (p, 0, h)* and $r_2(\phi)$ is the equation on the polar form relative to center of an ellipse, which describes a general distance from the center to the circumference of an ellipse by rotating the major axis angle (ϕ) and with semi diameters of major and minor axis *a* and *b* respectively of an ellipse. The form of $r_2(\phi)$ is given by:

$$r_2(\phi) = \frac{-p.\,b^2.\cos\phi + a.\,b\sqrt{(b.\cos\phi)^2 + (a^2-p^2)sin^2\phi}}{[(a.\sin\phi)^2 + (b.\cos\phi)^2]} \quad (20)$$

The geometrical notations of *a, b* and *h* are as shown in Fig. 2. By combining Eqs. (14-20), results in direct mathematical expression for the total efficiency subtended by an elliptical cylindrical detector, from an arbitrarily positioned radiating point source *S (p, 0, h)*.

Case II: $p = \rho = a$

Consider an elliptical cylindrical *(a, b, L)* detector and an arbitrarily positioned isotropic point source located at a distance *h* from the center of the detector top surface, as shown in Fig. (3). The efficiency of the detector with respect to a point source is given by as follows

$$\varepsilon = \frac{1}{4\pi}\int_0^{\pi}\left[\int_0^{\theta_5(\phi)} (1-e^{-\mu.d_5(\phi)})\sin\theta\, d\theta + \int_{\theta_5(\phi)}^{\theta_6(\phi)} (1-e^{-\mu.d_6(\phi)})\sin\theta\, d\theta\right] d\theta \quad (21)$$

where, $d_5(\theta)$ is the photon path length traveled through the detector active volume, it enters from the upper face and emerge from its base, which is represented by:

$$d_5(\theta) = \left(\frac{L}{\sin\theta_5(\phi)}\right) \quad (22)$$

while, $d_6(\theta)$ is the photon path length traveled through the detector active volume, it enters from the upper face and emerge from its side, which is represented by:

$$\mathrm{d}_6(\theta) = \left(\frac{\mathrm{r}_3(\phi)}{\sin\theta_6(\phi)}\right) - \left(\frac{\mathrm{h}}{\cos\theta_6(\phi)}\right) \quad (23)$$

where the polars $\theta_5(\phi)$ and $\theta_6(\phi)$ and the azimuthal ϕ angles are given by respectivly:

$$\theta_5(\phi) = \tan^{-1}\left(\frac{r_3(\phi)}{h+L}\right) \quad (24)$$

$$\theta_6(\phi) = \tan^{-1}\left(\frac{r_3(\phi)}{h}\right) \quad (25)$$

$$0 \le \phi \le 2\pi \quad (26)$$

where, *h* is the height from top surface of an elliptical cylindrical detector to an arbitrarily positioned radiating point source *S (p, 0, h)* and $r_3(\phi)$ is the equation on the polar form relative to center of an ellipse, which describes a general distance from the center to the circumference of an ellipse by rotating the major axis angle (ϕ) and with semi diameters of major and minor axis *a* and *b* respectively of an ellipse. The form of $r_3(\phi)$ is given by:

$$r_3(\phi) = \frac{2.\,a.\,b^2.\cos\phi}{[(a.\sin\phi)^2 + (b.\cos\phi)^2]} \quad (27)$$

The geometrical notations of *a, b* and *h* are as shown in Fig. 3. By combining Eqs. (21-27), results in direct mathematical expression for the total efficiency subtended by an elliptical cylindrical detector, from an arbitrarily positioned radiating point source *S (p, 0, h)*.

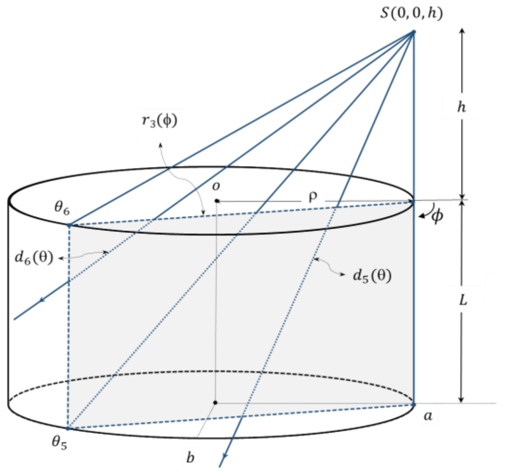

Fig. 3. Schematic view of an arbitrarily positioned radiating point source S (p, 0, h) located above the major axis of an elliptical cylindrical detector at a radial distance p = a and at a height h.

Case III: $p = \rho > a$

i. High source at height $h > L$ $\frac{r_4(\phi)}{(r_4(\phi)+r_5(\phi))(1-r_4(\phi))}$

Consider an elliptical cylindrical *(a, b, L)* detector and an arbitrarily positioned radiating point source located at a distance *h* from the center of the detector top surface, as shown in Fig. (4).

The efficiency of the detector with respect to a point source is given by as follows

$$\varepsilon = \frac{1}{4\pi}\int_0^{\phi_{max}} \left[\begin{array}{c} \int_{\theta_7(\phi)}^{\theta_8(\phi)} \left(1-e^{-\mu.d_7(\phi)}\right) \sin\theta \, d\theta \\ + \int_{\theta_8(\phi)}^{\theta_9(\phi)} \left(1-e^{-\mu.d_8(\phi)}\right) \sin\theta \, d\theta + \int_{\theta_9(\phi)}^{\theta_{10}(\phi)} \left(1-e^{-\mu.d_9(\phi)}\right) \sin\theta \, d\theta \end{array} \right] d\phi \qquad (28)$$

where $d_7(\theta)$ is the photon path length traveled through the detector active volume, it enters from its side and emerge from its base, which is represented by:

$$d_7(\theta) = \left(\frac{h+L}{\cos\theta_7(\phi)}\right) - \left(\frac{d_4(\phi)}{\sin\theta_7(\phi)}\right) \qquad (29)$$

where $d_8(\theta)$ is the photon path length traveled through the detector active volume, it enters from its the upper face and emerge from its base, which is represented by:

$$d_8(\theta) = \left(\frac{L}{\sin\theta_9(\phi)}\right) \qquad (30)$$

while, $d_9(\theta)$ is the photon path length traveled through the detector active volume, it enters from the upper face and emerge from its side, which is represented by:

$$d_9(\theta) = \left(\frac{r_4(\phi) + r_5(\phi)}{\sin\theta_{10}(\phi)}\right) - \left(\frac{h}{\cos\theta_{10}(\phi)}\right) \qquad (31)$$

where the polars $\theta_7(\phi)$, $\theta_8(\phi)$, $\theta_9(\phi)$ and $\theta_{10}(\phi)$ and the azimuthal ϕ angles are given by respectively.

$$\theta_7(\phi) = tan^{-1}\left(\frac{r_4(\phi)}{h+L}\right) \qquad (32)$$

$$\theta_8(\phi) = tan^{-1}\left(\frac{r_4(\phi)}{h}\right) \qquad (33)$$

$$\theta_9(\phi) = tan^{-1}\left(\frac{r_4(\phi)+r_5(\phi)}{h+L}\right) \qquad (34)$$

$$\theta_{10}(\phi) = tan^{-1}\left(\frac{r_4(\phi)+r_5(\phi)}{h}\right) \qquad (35)$$

and $0 \le \phi \le \phi_{max}$ with

$$\phi_{max} = a.tan\left(\sqrt{\frac{b^2}{\rho^2 - a^2}}\right) \qquad (36)$$

where *h* is the height from top surface of an elliptical cylindrical detector to an arbitrarily positioned radiating point source *S (p, 0, h)* and $r_4(\phi)$ is the distance from the point P(*p, 0 ,0*) into circumference of the ellipse, where P(*p, 0 ,0*) is located outside the elliptical cylindrical detector at distance *p* ($\rho > a$) from its center, as shown in Fig. (4), and $r_5(\phi)$ is the ellipse cord. $r_4(\phi)$ and $r_5(\phi)$ are given by:

$$r_4(\phi) = \left\{ \left[\frac{\rho.tan^2\phi + \frac{b}{a}\sqrt{tan^2\phi\,(a^2-\rho^2)+b^2}}{\left[(\tan(\phi))^2 + \frac{b^2}{a^2}\right]} - \rho \right]^2 [1+tan^2\phi] \right\}^{\frac{1}{2}} \qquad (37)$$

$$r_5(\phi) = 2\frac{b.\sqrt{[tan^2\phi\,(a^2-\rho^2)+b^2][1+tan^2\phi]}}{a.\left[tan^2(\phi) + \frac{b^2}{a^2}\right]} \qquad (38)$$

The geometrical notations of *a, b* and *h* are as shown in Fig. 4. By combining Eqs. (28-36), results in direct mathematical expression for the total efficiency subtended by an elliptical cylindrical detector, from an arbitrarily positioned radiating point source *S (p, 0, h)*.

ii. High source at height $h < L$ $\frac{r_4(\phi)}{(r_4(\phi)+r_5(\phi))(1-r_4(\phi))}$

Consider an elliptical cylindrical *(a, b, L)* detector and an arbitrarily positioned radiating point source located at a distance *h* from the center of the detector top surface, as shown in Fig. (5).

The efficiency of the detector with respect to point source is given by as follows

$$\varepsilon = \frac{2}{4\pi}\int_0^{\phi_{max}} \left[\begin{array}{c} \int_{\theta_7(\phi)}^{\theta_9(\phi)} \left(1-e^{-\mu.d_7(\phi)}\right) \sin\theta \, d\theta \\ + \int_{\theta_9(\phi)}^{\theta_8(\phi)} \left(1-e^{-\mu.d_{10}(\phi)}\right) \sin\theta \, d\theta + \int_{\theta_8(\phi)}^{\theta_{10}(\phi)} \left(1-e^{-\mu.d_9(\phi)}\right) \sin\theta \, d\theta \end{array} \right] d\phi \qquad (39)$$

where $d_7(\theta)$ is the photon path length traveled through the detector active volume, it enters from its side and emerge from its base, which is represented by:

$$d_7(\theta) = \left(\frac{h+L}{\cos\theta_7(\phi)}\right) - \left(\frac{r_4(\phi)}{\sin\theta_7(\phi)}\right) \qquad (40)$$

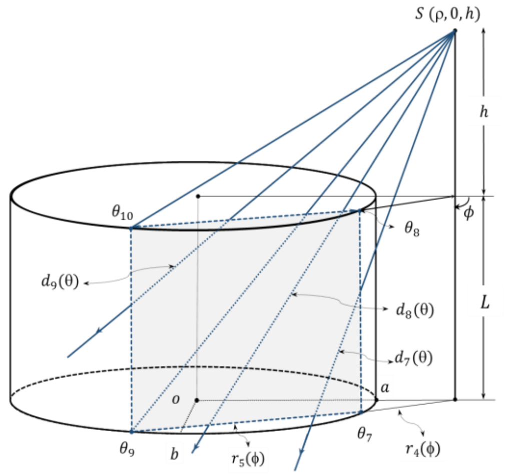

Fig. 4. Schematic view of an arbitrarily positioned radiating point source S (p, 0, h) located above the major axis of an elliptical cylindrical detector at a radial distance p>a, at a height h
$\boldsymbol{h > L \frac{r_4(\phi)}{(r_4(\phi)+r_5(\phi))(1-r_4(\phi))}}$).

while, $d_9(\theta)$ is the photon path length traveled through the detector active volume, it enters from the upper face and emerge from its side, which is represented by:

$$d_9(\theta) = \left(\frac{r_4(\phi) + r_5(\phi)}{\sin\theta_{10}(\phi)}\right) - \left(\frac{h}{\cos\theta_{10}(\phi)}\right) \tag{41}$$

while, $d_{10}(\theta)$ is the photon path length traveled through the detector active volume, it enters from the one and emerge from its opposite side, which is represented by:

$$d_{10}(\theta) = \left(\frac{r_5(\phi)}{\sin\theta_9(\phi)}\right) \tag{42}$$

where the polars $\theta_7(\phi)$, $\theta_8(\phi)$, $\theta_9(\phi)$ and $\theta_{10}(\phi)$ and the azimuthal ϕ angles are given by respectively:

$$\theta_7(\phi) = \tan^{-1}\left(\frac{r_4(\phi)}{h+L}\right) \tag{43}$$

$$\theta_8(\phi) = \tan^{-1}\left(\frac{r_4(\phi)}{h}\right) \tag{44}$$

$$\theta_9(\phi) = \tan^{-1}\left(\frac{r_4(\phi)+r_5(\phi)}{h+L}\right) \tag{45}$$

$$\theta_{10}(\phi) = \tan^{-1}\left(\frac{r_4(\phi)+r_5(\phi)}{h}\right) \tag{46}$$

and

$0 \leq \phi \leq \phi_{max}$

with

$$\phi_{max} = a.\tan\left(\sqrt{\frac{b^2}{\rho^2 - a^2}}\right) \tag{47}$$

where, h is the height from top surface of an elliptical cylindrical detector to an arbitrarily positioned radiating point source *S (p, 0, h)* and $r_4(\phi)$ is the distance from the point P (*p, 0, 0*) into circumference of the ellipse, where P(*p, 0 ,0*) is located outside the elliptical cylindrical detector at distance p ($\rho > a$) from its center, as shown in Fig. (4), and $r_5(\phi)$ is the ellipse cord. $r_4(\phi)$ and $r_5(\phi)$ are given by:

$$r_4(\phi) = \left\{\left[\frac{\rho.\tan^2\phi + \frac{b}{a}\sqrt{\tan^2\phi(a^2-\rho^2)+b^2}}{\left[\tan^2\phi + \frac{b^2}{a^2}\right]} - \rho\right]^2 [1+\tan^2\phi]\right\}^{\frac{1}{2}} \tag{48}$$

$$r_5(\phi) = 2\frac{b.\sqrt{[\tan^2\phi(a^2-p^2)+b^2][1+\tan^2\phi]}}{a.\left[\tan^2\phi + \frac{b^2}{a^2}\right]} \tag{49}$$

The geometrical notations of *a, b* and *h* are as shown in Fig. 5. By combining Eqs. (39-49), results in direct mathematical expression for the total efficiency subtended by an elliptical cylindrical detector, from an arbitrarily positioned radiating point source S (p, 0, h).

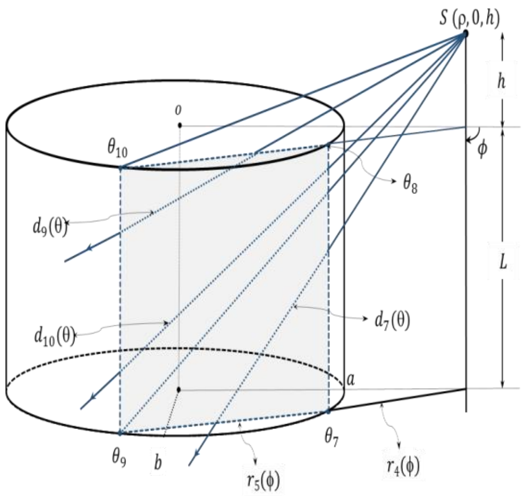

Fig.5. Schematic view of an arbitrarily positioned radiating point source S (p, 0, h) located above the major axis of an elliptical cylindrical detector with p>a, at a height h
$\boldsymbol{h < L \frac{r_4(\phi)}{(r_4(\phi)+r_5(\phi))(1-r_4(\phi))}}$).

3. Results

The total efficiency obtained for an energy range of 0.1 MeV to 10 MeV, the analytical formulae which derived for total efficiency shows a combination of the average path length covered by photon inside the detector active volume and the solid angle calculations. The total efficiencies for an elliptical NaI(Tl) cylindrical detector have been calculated and listed in Tables (1-5) relative to a several positions of an arbitrarily positioned radiating point source. Table 6 shows a systematic behavior, as move an arbitrarily positioned radiating point source at a radial distance for a fixed height, which shows the validity of equations. Also the validity of the derived analytical expressions was successfully confirmed by the comparison with some published data.

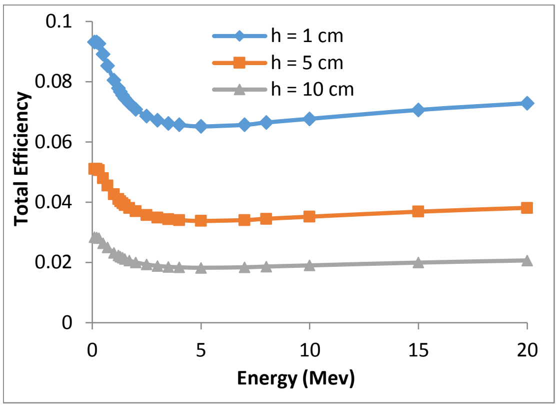

Fig.6. Measured total efficiency values for an arbitrarily positioned point source which is located at radial distances (ρ = 0 cm) and at different heights (h = 1 cm, 5 cm and 10 cm) above the major axis (a = 7.62 cm) of an elliptical cylindrical detector.

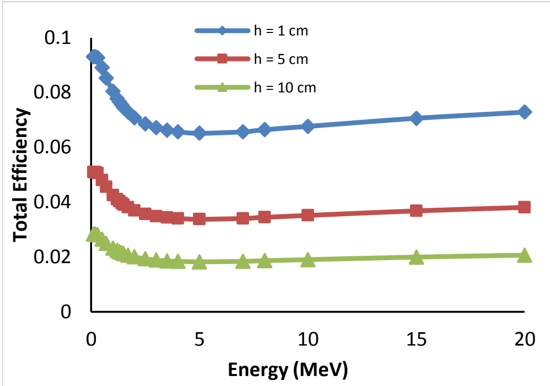

Fig.7. Measured total efficiency values for an arbitrarily positioned point source which is located at radial distances (ρ = 0.1 cm) and at different heights (h = 1 cm, 5 cm and 10 cm) above the major axis (a = 7.62 cm) of an elliptical cylindrical detector.

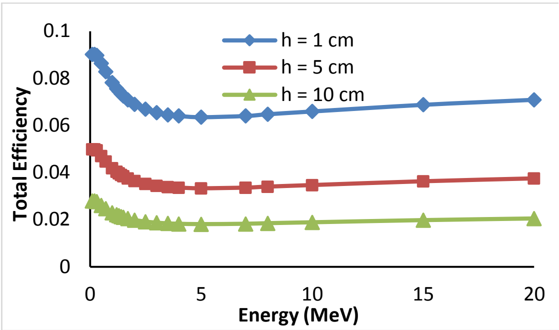

Fig.8. Measured total efficiency values for an arbitrarily positioned point source which is located at radial distances (ρ = 2 cm) and at different heights (h = 1 cm, 5 cm and 10 cm) above the major axis (a = 7.62 cm) of an elliptical cylindrical detector.

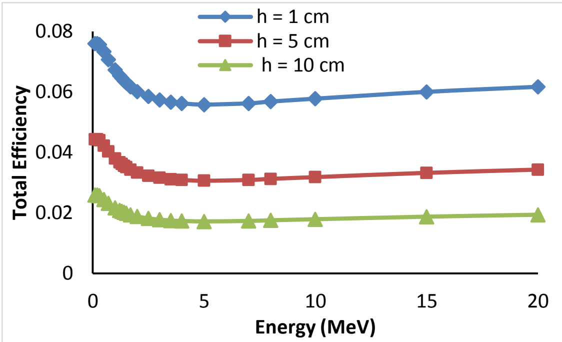

Fig.9. Measured total efficiency values for an arbitrarily positioned point source which is located at radial distances (ρ = 5 cm) and at different heights (h = 1 cm, 5 cm and 10 cm) above the major axis (a = 7.62 cm) of an elliptical cylindrical detector.

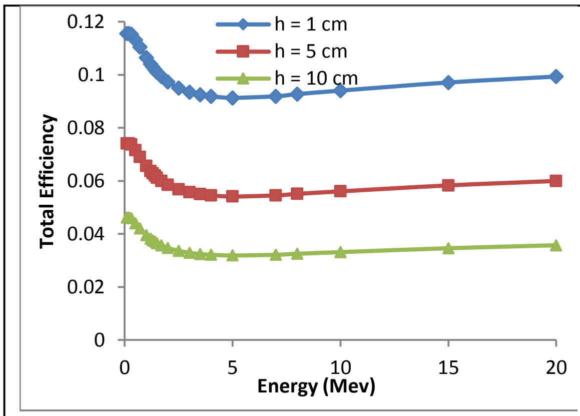

Fig.10. Measured total efficiency values for an arbitrarily positioned point source which is located at radial distances (ρ = 7.62 cm) and at different heights (h = 1 cm, 5 cm and 10 cm) above the major axis (a = 7.62 cm) of an elliptical cylindrical detector.

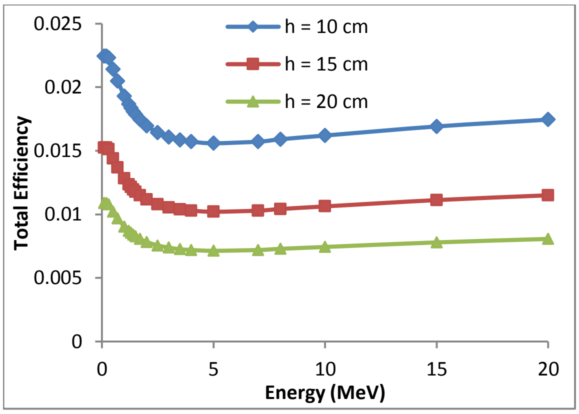

Fig. 11. Measured total efficiency values for an arbitrarily positioned point source which is located at radial distances (ρ = 8 cm) and at different heights (h = 10 cm, 15 cm and 20 cm) above the major axis (a = 7.62 cm) of an elliptical cylindrical detector.

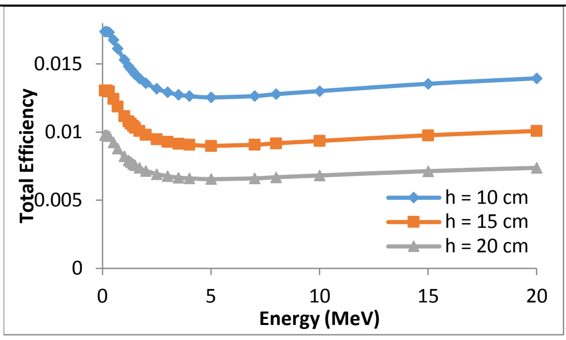

Fig.12. Measured total efficiency values for an arbitrarily positioned point source which is located at radial distances (ρ = 10 cm) and at different heights (h = 10 cm, 15 cm and 20 cm) above the major axis (a = 7.62 cm) of an elliptical cylindrical detector.

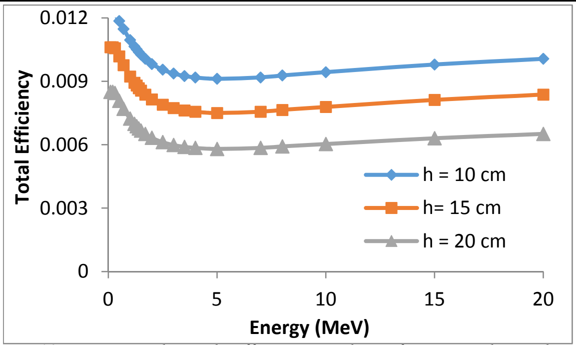

Fig.13. Measured total efficiency values for an arbitrarily positioned point source which is located at radial distances (ρ = 12 cm) and at different heights (h = 10 cm, 15 cm and 20 cm) above the major axis (a = 7.62 cm) of an elliptical cylindrical detector

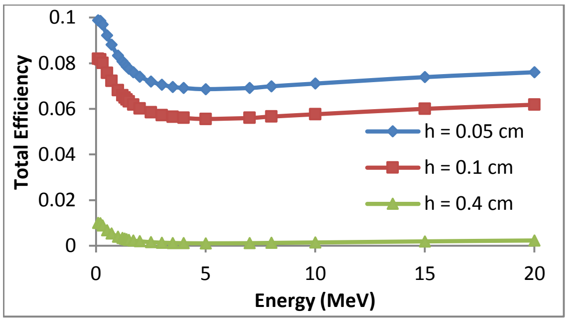

Fig.14. Measured total efficiency values for an arbitrarily positioned point source which is located at radial distances (ρ= 8 cm) and at different heights (h = 0.05 cm, 0.1 cm, 0.4 cm) above the major axis (a = 7.62 cm) of an elliptical cylindrical detector.

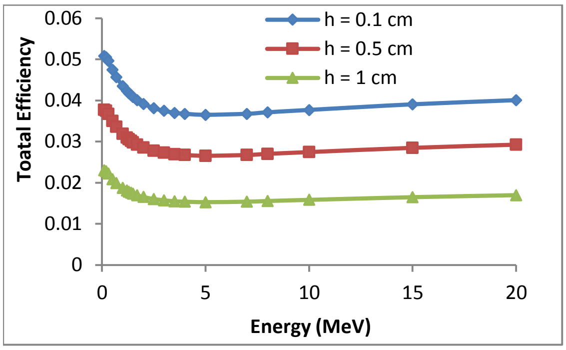

Fig.15. Measured total efficiency values for an arbitrarily positioned point source which is located at radial distances (ρ= 10 cm) and at different heights (h = 0.1 cm, 0.5 cm, 1 cm) above the major axis (a = 7.62 cm) of an elliptical cylindrical detector.

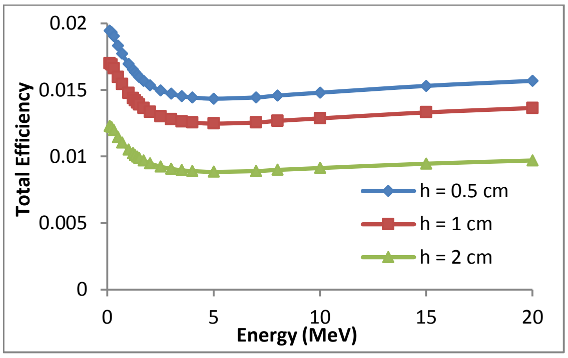

Fig.16. Measured total efficiency values for an arbitrarily positioned point source which is located at radial distances (ρ= 15 cm) and at different heights (h = 0.5 cm, 1 cm, 2 cm) above the major axis (a = 7.62 cm) of an elliptical cylindrical detector

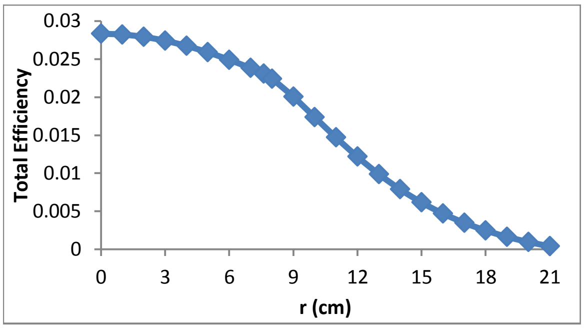

Fig.17. Variation of total efficiency values for an arbitrarily positioned point source with variation of radial distances which is located at fixed height (h = 10 cm) above the major axis of an elliptical cylindrical detector.

4. Conclusions

Direct mathematical expressions to calculate total efficiency of an elliptical cylindrical NaI(Tl) detector have been derived in the case of an arbitrarily positioned radiating point source. This work gives a new step-up in the γ-ray spectroscopy, where it calculates the total efficiency for absolute γ source with an elliptical cylindrical detector.

Table 1. The measured values of total efficiency for an arbitrarily positioned radiating point source S (ρ, 0, h) which is located at radial distances (ρ = 0 cm) and at heights (h = 1 cm, 5 cm and 10 cm) above the center of an elliptical cylindrical detector.

E (Mev) \ ε	ρ = 0, h: 1 cm	5 cm	10 cm
0.1	0.093124	0.051011	0.02835
0.2	0.093121	0.051007	0.028346
0.3	0.092662	0.050594	0.028055
0.5	0.089079	0.048018	0.026431
0.7	0.085303	0.045571	0.024965
1	0.080472	0.042604	0.023231
1.2	0.077843	0.041038	0.022329
1.3	0.076644	0.040333	0.021926
1.4	0.075518	0.039675	0.021551
1.5	0.074491	0.039079	0.021211
1.7	0.072837	0.038125	0.020671
2	0.070827	0.036976	0.020022
2.5	0.068617	0.035725	0.019318
3	0.067187	0.034921	0.018868
3.5	0.066187	0.034361	0.018555
4	0.065673	0.034074	0.018395
5	0.065149	0.033782	0.018232
7	0.065673	0.034074	0.018395
8	0.06644	0.034503	0.018634
10	0.067672	0.035193	0.019021
15	0.070615	0.036856	0.019954
20	0.072837	0.038125	0.020671

Table 2. The measured values of total efficiency for an arbitrarily positioned radiating point source S (ρ, 0, h) which is located at radial distances (ρ = 7.62 cm) and at heights (h = 1 cm, 5 cm and 10 cm) above the major axis (a = 7.62 cm) of an elliptical cylindrical detector.

E (MeV) \ ε	ρ = a, h: 1 cm	5 cm	10 cm
0.1	0.115535065	0.0740567688	0.046228173
0.2	0.115533586	0.0740548911	0.046225917
0.3	0.115311948	0.0737860992	0.045952674
0.5	0.113158059	0.0715271014	0.044021928
0.7	0.110418747	0.0689837919	0.042058959
1	0.106437743	0.0655878099	0.039588014
1.2	0.104087763	0.0636896265	0.038255338
1.3	0.102977605	0.0628139133	0.037649639
1.4	0.101914071	0.0619860766	0.03708176
1.5	0.100927727	0.0612272734	0.036564974
1.7	0.099306265	0.0599970344	0.03573415
2	0.097282516	0.0584886734	0.034726379
2.5	0.09499433	0.0568154842	0.033621149
3	0.09347971	0.0557249159	0.032907321
3.5	0.092405692	0.0549591482	0.032408971
4	0.091848391	0.0545641484	0.032152803
5	0.091277091	0.0541608411	0.031891855
7	0.091848391	0.0545641484	0.032152803
8	0.092679204	0.0551535871	0.032535291
10	0.09399711	0.0560960202	0.033149673
15	0.097066155	0.0583290588	0.034620392
20	0.099306265	0.0599970344	0.03573415

Table 3. The measured values of total efficiency for an arbitrarily positioned radiating point source S (ρ, 0, h) which is located at radial distances (ρ = 0.1 cm, 2 cm and 5 cm) and at different heights (h = 1 cm, 5 cm and 10 cm) above the major axis (a = 7.62 cm) of an elliptical cylindrical detector.

ε / E (MeV)	ρ = 0.1 cm			ρ = 2 cm			ρ = 5 cm		
	h			h			h		
	1 cm	5 cm	10 cm	1 cm	5 cm	10 cm	1 cm	5 cm	10 cm
0.1	0.093117	0.051009	0.028349	0.090191	0.049862	0.027938	0.075945	0.044304	0.025909
0.2	0.093114	0.051005	0.028345	0.090188	0.049858	0.027935	0.075942	0.044302	0.025907
0.3	0.092654	0.050591	0.028054	0.089746	0.049469	0.027657	0.075628	0.044029	0.025692
0.5	0.089072	0.048015	0.026430	0.086363	0.047020	0.026088	0.073263	0.042178	0.024387
0.7	0.085297	0.045569	0.024964	0.082792	0.044675	0.024661	0.070682	0.040311	0.023151
1	0.080466	0.042602	0.023230	0.078204	0.041814	0.022967	0.067263	0.037963	0.021651
1.2	0.077838	0.041036	0.022329	0.075700	0.040300	0.022084	0.065354	0.036696	0.020860
1.3	0.076639	0.040331	0.021925	0.074556	0.039617	0.021689	0.064474	0.03612	0.020503
1.4	0.075512	0.039673	0.021550	0.073481	0.038979	0.021320	0.063642	0.035579	0.020170
1.5	0.074486	0.039077	0.021211	0.072500	0.038401	0.020988	0.062879	0.035088	0.019869
1.7	0.072833	0.038124	0.020670	0.070918	0.037476	0.020457	0.061641	0.034297	0.019386
2	0.070822	0.036975	0.020022	0.068992	0.036359	0.019819	0.060122	0.033337	0.018805
2.5	0.068612	0.035723	0.019318	0.066873	0.035142	0.019127	0.058436	0.032284	0.018171
3	0.067182	0.034919	0.018867	0.065500	0.034359	0.018684	0.057336	0.031604	0.017764
3.5	0.066183	0.034360	0.018555	0.064539	0.033814	0.018376	0.056563	0.031129	0.017481
4	0.065669	0.034073	0.018395	0.064045	0.033534	0.018219	0.056164	0.030884	0.017335
5	0.065145	0.033781	0.018232	0.063541	0.033250	0.018058	0.055757	0.030636	0.017187
7	0.065669	0.034073	0.018395	0.064045	0.033534	0.018219	0.056164	0.030884	0.017335
8	0.066436	0.034502	0.018634	0.064783	0.033952	0.018454	0.056759	0.031249	0.017552
10	0.067668	0.035192	0.019020	0.065966	0.034624	0.018834	0.057710	0.031835	0.017902
15	0.070611	0.036854	0.019954	0.068790	0.036242	0.019753	0.059961	0.033236	0.018744
20	0.072833	0.038124	0.020670	0.070918	0.037476	0.020457	0.061641	0.034297	0.019386

Table 4. The measured values of total efficiency for an arbitrarily positioned radiating point source S (ρ, 0, h) which is located at radial distances (ρ= 8 cm, 10 cm and 12 cm) and at heights (h = 10 cm, 15 cm and 20 cm) above the major axis (a = 7.62 cm) of an elliptical cylindrical detector.

ε	**ρ = 8 cm**			**ρ = 10 cm**			**ρ = 12 cm**		
	h			***h***			***h***		
***E* (MeV)**	**10 cm**	**15 cm**	**20 cm**	**10 cm**	**15 cm**	**20 cm**	**10 cm**	**15 cm**	**20 cm**
0.1	0.022446	0.015239	0.010896	0.017395	0.013069	0.009797	0.01218777	0.01061427	0.00849969
0.2	0.022445	0.015238	0.010895	0.017395	0.013068	0.009796	0.01218770	0.01061402	0.00849930
0.3	0.022321	0.015124	0.010799	0.017332	0.012992	0.009723	0.01216019	0.01056721	0.00844644
0.5	0.021419	0.014395	0.010223	0.016774	0.012445	0.009250	0.01185438	0.01018242	0.00807443
0.7	0.020489	0.013688	0.009687	0.016146	0.011886	0.008793	0.01147705	0.00976733	0.00770071
1	0.019308	0.012822	0.009042	0.015314	0.011183	0.008234	0.01095335	0.00923007	0.00723437
1.2	0.018668	0.012363	0.008703	0.014852	0.010804	0.007937	0.01065473	0.00893578	0.00698428
1.3	0.018377	0.012155	0.008551	0.014639	0.010632	0.007803	0.01051592	0.00880120	0.00687089
1.4	0.018104	0.011961	0.008410	0.014439	0.010471	0.007678	0.01038416	0.00867460	0.00676473
1.5	0.017855	0.011785	0.008281	0.014255	0.010324	0.007565	0.01026298	0.00855906	0.00666824
1.7	0.017455	0.011504	0.008076	0.013958	0.010088	0.007383	0.01006574	0.00837270	0.00651334
2	0.016968	0.011163	0.007829	0.013595	0.009802	0.007164	0.00982275	0.00814570	0.00632579
2.5	0.016434	0.010792	0.007561	0.013194	0.009488	0.006925	0.00955187	0.00789572	0.00612051
3	0.016089	0.010553	0.007389	0.012932	0.009285	0.006771	0.00937465	0.00773368	0.00598813
3.5	0.015848	0.010387	0.007269	0.012749	0.009144	0.006664	0.00924991	0.00762032	0.00589582
4	0.015724	0.010302	0.007208	0.012655	0.009072	0.006609	0.00918548	0.00756197	0.00584839
5	0.015597	0.010215	0.007145	0.012559	0.008998	0.006553	0.00911964	0.00750248	0.00580010
7	0.015724	0.010302	0.007208	0.012655	0.009072	0.006609	0.00918548	0.00756197	0.00584839
8	0.015909	0.010429	0.007300	0.012796	0.009180	0.006691	0.00928161	0.00764907	0.00591921
10	0.016206	0.010634	0.007447	0.013021	0.009354	0.006823	0.00943501	0.00778874	0.00603306
15	0.016917	0.011128	0.007804	0.013557	0.009772	0.007141	0.00979696	0.00812177	0.00630608
20	0.017455	0.011504	0.008076	0.013958	0.010088	0.007383	0.01006574	0.00837270	0.00651334

Table 5. The measured values of total efficiency for an arbitrarily positioned radiating point source S (ρ, 0, h) which is located at radial distances (ρ= 8 cm, 10 cm and 15 cm) and at heights (h = 0.05 cm, 0.1 cm, 0.4 cm, 0.5 cm, 1 cm and 2 cm) above the major axis (a = 7.62 cm) of an elliptical cylindrical detector.

ε	ρ = 8 cm			ρ = 10 cm			ρ = 15 cm		
	h			***h***			***h***		
E (MeV)	**0.05 cm**	**0.1 cm**	**0.4 cm**	**0.1 cm**	**0.5 cm**	**1 cm**	**0.5 cm**	**1 cm**	**2 cm**
0.1	0.098864	0.081904	0.0100342	0.0507931	0.03775	0.0230118	0.0194475	0.0170016	0.012284
0.2	0.09854	0.081596	0.0098158	0.0505086	0.0375088	0.0228157	0.0193455	0.0169051	0.012202
0.3	0.096939	0.080112	0.0089029	0.0496059	0.0367492	0.0222131	0.019046	0.0166304	0.011975
0.5	0.092144	0.075788	0.0067629	0.0474317	0.034972	0.020873	0.0183255	0.0159803	0.011456
0.7	0.088166	0.072279	0.0053584	0.0456662	0.0335691	0.0198703	0.0177161	0.0154376	0.011039
1	0.083393	0.068132	0.00397	0.0435065	0.0318848	0.0187129	0.0169474	0.0147578	0.010526
1.2	0.080849	0.065945	0.0033349	0.0423334	0.030981	0.0181081	0.0165218	0.014383	0.010247
1.3	0.079694	0.064956	0.0030673	0.0417958	0.0305688	0.0178356	0.0163252	0.0142101	0.010119
1.4	0.078611	0.064031	0.0028273	0.0412888	0.0301812	0.017581	0.016139	0.0140464	0.009998
1.5	0.077625	0.06319	0.0026176	0.0408247	0.0298273	0.0173498	0.0159678	0.0138961	0.009887
1.7	0.076036	0.06184	0.0022966	0.0400728	0.0292555	0.016979	0.0156894	0.0136518	0.009708
2	0.074103	0.060203	0.0019324	0.0391508	0.0285569	0.0165299	0.0153461	0.0133509	0.009487
2.5	0.071975	0.058407	0.0015626	0.0381264	0.0277837	0.0160376	0.0149624	0.0130149	0.009242
3	0.070594	0.057245	0.0013391	0.0374571	0.0272802	0.0157195	0.0147107	0.0127947	0.009082
3.5	0.069628	0.056434	0.0011899	0.0369864	0.0269266	0.0154972	0.0145331	0.0126393	0.008969
4	0.06913	0.056016	0.0011154	0.0367432	0.0267442	0.0153828	0.0144412	0.012559	0.00891
5	0.068622	0.05559	0.0010409	0.0364947	0.026558	0.0152663	0.0143472	0.0124768	0.008851
7	0.06913	0.056016	0.0011154	0.0367432	0.0267442	0.0153828	0.0144412	0.012559	0.00891
8	0.069873	0.056639	0.0012272	0.037106	0.0270164	0.0155535	0.0145782	0.0126788	0.008998
10	0.071064	0.05764	0.0014137	0.037685	0.0274515	0.0158275	0.0147965	0.0128697	0.009136
15	0.0739	0.060031	0.0018956	0.0390531	0.0284831	0.0164827	0.0153096	0.0133189	0.009464
20	0.076036	0.06184	0.0022966	0.0400728	0.0292555	0.016979	0.0156894	0.0136518	0.009708

Table 6. Variation of Total Efficiency with the radial distance (ρ) for an arbitrarily positioned radiating point source which is located at fixed heights (h = 10 cm) above the major axis (a = 7.62 cm) of an elliptical cylindrical detector.

ρ (cm)	Total Efficiency	ρ (cm)	Total Efficiency
0	0.028349818020	**11**	0.014712739007
1	0.028246163543	**12**	0.012187734581
2	0.027938323624	**13**	0.009905776158
3	0.027435500528	**14**	0.007902198219
4	0.026752479720	**15**	0.006179815226
5	0.025908835727	**16**	0.004722125600
6	0.024927889304	**17**	0.003502745740
7	0.023835487735	**18**	0.002491644775
7.62	0.023114086321	**19**	0.001658917915
8	0.022445747079	**20**	0.000976839955
9	0.020073436823	**21**	0.000420749742
10	0.017395154433		

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