Ankara University
Institute of Nuclear Sciences

Journal of Nuclear Sciences

Ground-state properties of some N=Z nuclei

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ABSTRACT

The ground-state properties of ^{64}Ge, ^{68}Se, ^{72}Kr and ^{76}Sr (N=Z) nuclei have been investigated by using Hartree-Fock-Bogolibov (HFB) method with Sly4 Skyrme forces, Relativistic Mean Field (RMF) model with NL3 and recently proposed DEFNE interaction parameters sets. For determination of ground-state, axially deformed shape and quadrupole moment constrained calculations have been employed in RMF model. The results of the present study have been compared with each other and available experimental data in the literature. The ground-state binding energies, neutron, proton and charge radii, quadrupole moment deformation parameters of these nuclei have been calculated. Furthermore, neutron skin thickness of the above mentioned nuclei as a function of deformation parameter have been obtained and discussed in detail.

Keywords: Nuclear properties, Neutron skin thickness, HFB method, RMF model, N=Z nuclei

1. Introduction

Atomic nuclei at or around the N=Z line have been of great interest in nuclear physics. In this type of nuclei protons and neutrons occupy same orbits. They form a unique laboratory for studying various phenomena such as the interplay of T=0 and 1 states close to the ground state, the shape coexistence along the N=Z line and the role of neutron–proton pairing correlations. In addition, these nuclei lie along the explosive rp-process nuclear synthesis pathway [1]. From theoretical side one of the main aim of research in nuclear physics is to describe the ground-state properties of all nuclei in the periodic table with one nuclear model. Due to the fact that the strong interaction is not yet fully understood and to the numerical difficulties in handling the nuclear many-body problem, relativistic and nonrelativistic phenomenological descriptions have received much attention for describing the ground-state properties of nuclei. Because of this reason Hartree-Fock-Bogoliubov (HFB) method [2,3] and Relativistic Mean Field (RMF) model [4-8] have been used successfully for describing ground-state properties of nuclei during the last decades. Therefore, main interest of the study is investigation of

some ground-state nuclear properties of ^{64}Ge, ^{68}Se, ^{72}Kr and ^{76}Sr nuclei.

In the present study, HFB method and RMF model have been employed for ^{64}Ge, ^{68}Se, ^{72}Kr and ^{76}Sr (N=Z) nuclei by considering axially deformed case. Because these nuclei do not have shell closure which means that these nuclei possibly have deformed shape in their ground-states. The binding energies, radii and quadrupole deformation parameters of the interested nuclei calculated by using both HFB method and RMF model. The obtained results are in agreement with the available data. Furthermore, in the case of RMF model applications for these nuclei, quadrupole constrained RMF calculations have been used. This application provides ground-state properties of nuclei as a function of deformation parameter (β_2). By using this method, neutron skin thickness of ^{64}Ge, ^{68}Se, ^{72}Kr and ^{76}Sr nuclei has been estimated as a function of fixed β_2. The effect of quadrupole deformation parameter has also been discussed on neutron skin thickness. Additionally, the obtained potential energy curves (PECs) have been used for determination of the ground-state shape of these nuclei.

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2. Material and Methods

In order to save the space, the full descriptions of HFB and RMF formalism is not given in this paper because of the complexity of phenomenology of both models. Therefore, it is suggest that the reader can get more information about these models from the References [2-8]. However, for the simplicity, it should be noted that, in the HFB method, many properties of nuclei can be described in terms of a model of independent particles moving in an average potential whose space dependence closely follows the matter distribution. In our axially deformed HFB calculations, the recipe of Stoitsov et al. [9] has been followed and effective Sly4 Skyrme forces used. The number of shells taken into account is 20. The ground-state properties of ^{64}Ge, ^{68}Se, ^{72}Kr and ^{76}Sr nuclei in HFB method have been calculated both for oblate and prolate shape. The shape of nuclei which gives lowest binding energy for nuclei has been taken as the ground-state of nuclei.

Nucleons interact with each other via exchange of various mesons in the RMF model. It starts with a Lagrangian density, includes terms related with free nucleons, free mesons and nucleon-meson interactions. This Lagrangian density includes some adjustable parameters such as masses of nucleons, mases of mesons, coupling constants of nucleon-meson interactions and coupling constants of self-interactions of mesons. These parameters are called as RMF parameters and three types of RMF model parameters can be found in literature. In this study, non-linear version RMF model with NL3 [10] was used, together with the recently proposed DEFNE parameters [11]. For determination of the ground-state properties of ^{64}Ge, ^{68}Se, ^{72}Kr and ^{76}Sr nuclei in RMF model quadrupole moment constrained calculations has been employed in such a way that for fixed quadrupole moment and binding energies of nuclei calculated. Thus PECs of the interested nuclei were also calculated. One can easily determine the ground-state of nuclei by investigating the lowest point of PECs. 12 and 20 shells have been considered for fermionic and bosonic expansion in the present study. The numerical explanation of RMF model can be found in Ref. [12].

3. Results and Discussion

The calculated ground state binding energies of ^{64}Ge, ^{68}Se, ^{72}Kr and ^{76}Sr nuclei by using HFB method with Sly4 method, RMF model with NL3 and DEFNE parameter set generated recently by our working group are listed in Table 1. Also the predictions of finite-range-droplet-model (FRDM) and the experimental data are listed for comparison. The binding energy values of considered nuclei calculated with HFB method and RMF model are close to each other and in agreement with experimental data. According to the calculated ground-state binding energies, DEFNE parameter set results are generally closer to the experimental values than those NL3 parameters for several N=Z nuclei except for ^{64}Ge. However closest predictions for binding energies are those of FRDM when compared with those of HFB method and RMF model. It should be noted that the success of FRDM resulted from the fitting of many parameters by using experimental data.

In order to determine the ground-state shape of nuclei, the PEC is an important tool. In Fig. 1, the calculated PECs of ^{64}Ge, ^{68}Se, ^{72}Kr and ^{76}Sr nuclei in RMF model with NL3 and DEFNE interactions are shown as a function of β_2. In this figure lowest binding energy has been taken as a reference. Positive value of β_2 is related with prolate shape while negative value of β_2 corresponds with oblate shape. As can be seen in the PECs of Fig. 1, ^{64}Ge and ^{76}Sr nuclei have prolate shape whereas ^{68}Se and ^{72}Kr have oblate shape. The β_2 quadrupole deformation parameters of the nuclei have also been calculated. Other than ^{68}Se, the nuclei are found to be compatible with each other and give the same ground state shapes. In the case of ^{68}Se, HFB and RMF calculations suggest that the nuclei are oblate shaped in its ground state whereas FRDM result indicates it as prolate. According to the common literature results [15], the nucleus has oblate shape as indicated in the present work (Table 2).

Table 1. *The ground state binding energy (in units of MeV) of some N=Z nuclei calculated by the HFB method with the Skyrme force Sly4 and RMF model with DEFNE and NL3 parameters. The predictions of FRDM [13] and experimental data [14] are also shown for comparison.*

	Ground state binding energy (MeV)				
Isotope	**HFB**	**RMF-DEFNE**	**RMF-NL3**	**FRDM**	**Experimental**
^{64}Ge	543.899	528.841	538.433	544.61	545.882
^{68}Se	575.410	573.974	583.852	575.44	576.463
^{72}Kr	604.381	614.592	623.259	606.72	606.911
^{76}Sr	635.171	652.140	657.335	638.26	637.936

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Table 2. *The calculated quadrupole deformation parameter β_2 for the ground-state of ^{64}Ge, ^{68}Se, ^{72}Kr and ^{76}Sr nuclei. The FRDM predictions [13] are shown for comparison.*

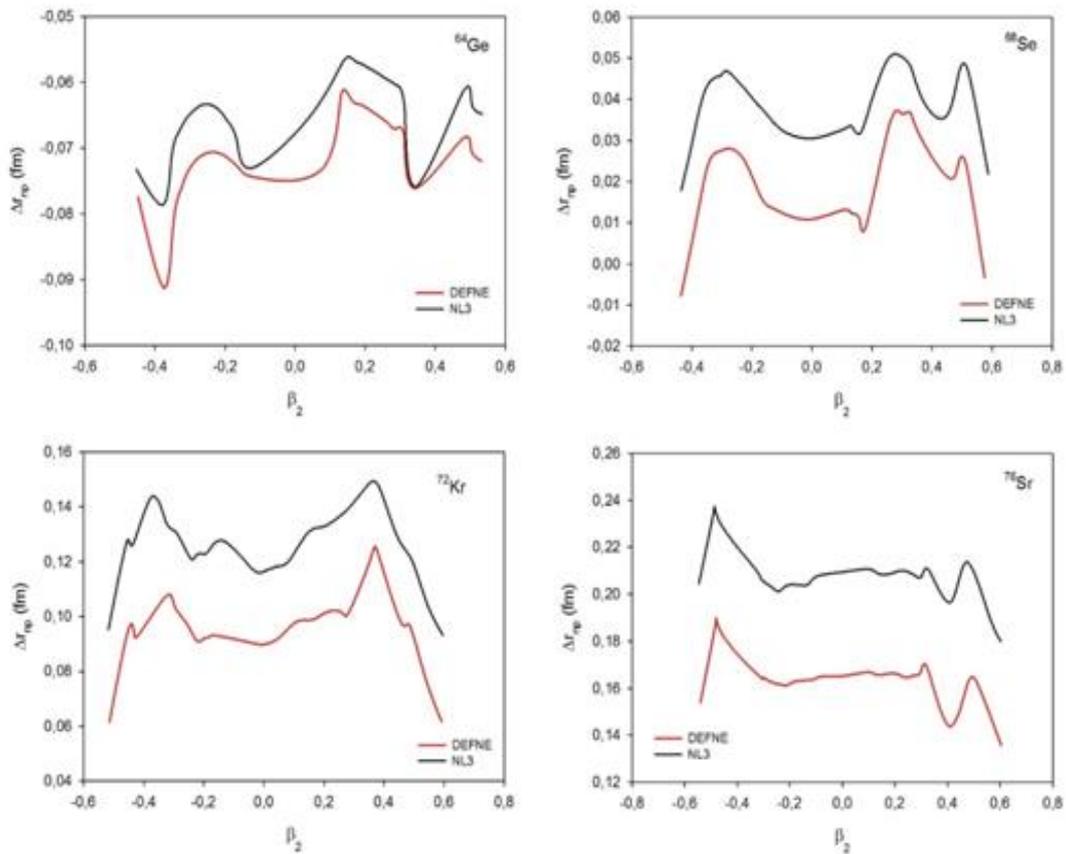| Quadrupole deformation parameter β_2				
Isotope	**HFB**	**RMF-DEFNE**	**RMF-NL3**	**FRDM**
^{64}**Ge**	0.183	0.237	0.239	0.207
^{68}**Se**	-0.225	-0.295	-0.300	0.233
^{72}**Kr**	-0.174	-0.202	-0.206	-0.366
^{76}**Sr**	0	0.242	0.190	0.402

Fig. 1 *The calculated PECs of ^{64}Ge, ^{68}Se, ^{72}Kr and ^{76}Sr nuclei in the RMF model.*

An important quantity of nuclear properties of nuclei is root mean square (rms) charge radii (r_c). Therefore, rms charge radii of the interested nuclei have been calculated in both HFB method and RMF models. The calculated charge radii of the nuclei are listed in Table 3. Also, neutron and proton radii are listed in the Table 3. It should be noted that there is no experimental data for these nuclei in literature. The calculated results from HFB and RMF models are in agreement with each other within maximum 0.5% differences.

Neutron skin thickness of nuclei has an important role in theoretical investigations of stellar studies. It can be obtained by using neutron and proton radii of nuclei in the formula $\Delta r_{np} = r_n - r_p$. The calculated Δr_{np} by using DEFNE and NL3 effective interactions in RMF model are shown in Fig. 2. As it is expected, Δr_{np} is increased with addition of nucleons.
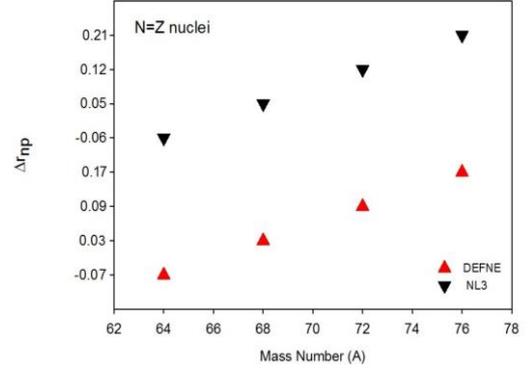

Fig. 2 *Neutron skin thickness of considered N=Z nuclei within the framework of non-linear RMF model.*

Table 3. *The calculated rms charge radii of the interested N=Z nuclei.*

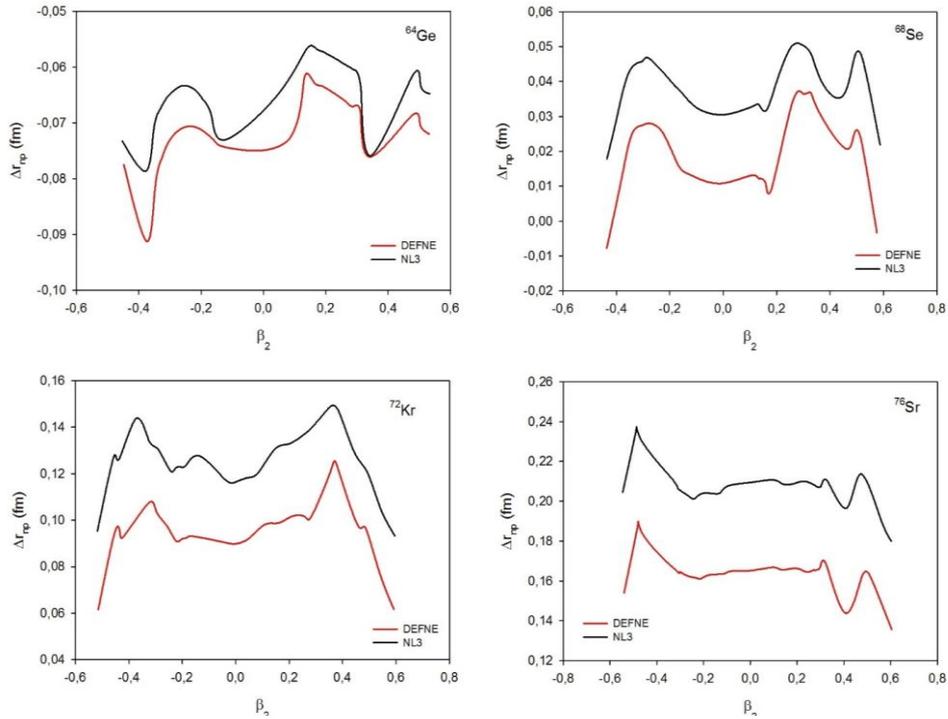|	The calculated rms charge radii (fm)			
Isotope	**HFB**	r_c **(DEFNE/NL3)**	r_n **(DEFNE/NL3)**	r_p **(DEFNE/NL3)**
^{64}Ge	4.00	3.98/3.97	3.83/3.83	3.90/3.89
^{68}Se	4.10	4.05/4.03	3.98/3.99	3.95/3.95
^{72}Kr	4.16	4.05/4.03	4.06/4.08	3.97/3.95
^{76}Sr	4.20	4.07/4.05	4.16/4.18	3.99/3.97

Fig. 3 *The calculated neutron skin thickness of considered N=Z nuclei in RMF model with DEFNE and NL3 interactions as a function of deformation parameter.*

Furthermore, an investigation of effect of quadrupole deformation parameter on neutron skin thickness can be an interesting study. In Fig. 3, Δr_{np} values of ^{64}Ge, ^{68}Se, ^{72}Kr and ^{76}Sr nuclei calculated by using DEFNE and NL3 parameters sets are shown as a function of β_2. General tendency of curves is same both for DEFNE and NL3 interactions. However, the NL3 parameter set predictions for neutron skin thickness are somehow higher than those of NL3.

4. Conclusion

HFB and RMF models have been used for investigation of the ground-state properties of even-even N=Z nuclei with proton numbers Z=32–38. The binding energy, rms charge radius, proton and neutron radius and neutron skin thickness of the nuclei are reproduced well by using the methods. In particular, the DEFNE interaction in the frame of RMF model is found to be a successful interaction for describing of ground-state properties of the nuclei. The ground-state shapes of ^{64}Ge, ^{68}Se, ^{72}Kr and ^{76}Sr nuclei have been determined by using the PECs of the nuclei by applying quadrupole moment constrained RMF calculations.

Conflict of Interest

The authors have no conflict of interest.

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