Volume 4, No. 1 July 2017 p-ISSN: 2147-7736, e-ISSN:2148-3981

Ankara University
Institute of Nuclear Sciences

Journal of Nuclear Sciences

Owner on behalf of Institute of Nuclear Sciences,
Ankara University,
Director

Niyazi MERİÇ, Ph.D.

http://dergipark.gov.tr/ankujns

Hosted by Ankara University

Journal of Nuclear Sciences

p-ISSN: 2147-7736, e-ISSN:2148-3981

Journal homepage: http://dergipark.gov.tr/ankujns

DOI: 10.1501/nuclear.2023.21

On the Role of Strong Coupling Constant and Nucleons in Understanding Nuclear Stability and Binding Energy

U.V.S. Seshavatharam[1*], S. Lakshminarayana[2]

ORCID: 0000-0002-1695-6037,-----

[1]Honorary faculty, I-SERVE, Survey no-42, Hitech city, Hyderabad-84, Telangana, India
[2]Dept. of Nuclear Physics, Andhra University, Visakhapatnam-03,AP, India

Received 11.07.2017; received in revised form 11.08.2017; accepted 14.08.2017

ABSTRACT

In this study, we dealt with beta stability line and nuclear binding energy in a simplified semi empirical approach by taking into account nucleon and electron rest masses and with reference to nuclear charge radius and strong coupling constant. In this context, we have considered $\left(1/\alpha_s\right)\left(e^2/8\pi\epsilon_0 R_0\right) \cong 4.86\ \mathrm{MeV}$ as a single nuclear binding energy coefficient or potential.

Keywords: Nucleon, Nuclear binding energy, Beta stability line, Strong coupling, Semi-empirical mass formula, Liquid drop model

1. Introduction

In nuclear physics, the semi-empirical mass formula [1-3] is used to approximate the mass and various other properties of an atomic nucleus. As its name suggests, it is based partly on theory and partly on empirical measurements. Although refinements have been made to the coefficients over the years, the structure of the formula remains the same today. The simple semi empirical mass formula (SEMF) constitutes of five different energy coefficients and 5 different energy terms. The major drawback of SEMF is that, it does not throw light on the implementation of strong interaction concepts and strong coupling constant [4,5] in understanding nuclear binding energy. In this study, we attempt:

1) To propose a very simple and direct relation for understanding beta stability line [5, 6, 7] with reference to neutron, proton and electron rest masses,

2) To propose a very simple semi empirical relation for fitting the nuclear binding energy with only one energy constant i.e. 4.86 MeV, which is assumed to be connected with nuclear charge radius $R_0 \cong 1.25$ fm and strong coupling constant $\alpha_s \cong 0.1186$,

3) To divide the nuclear binding energy scheme into 3 steps:

a. To understand the binding energy at stable mass number of Z,
b. To understand the binding energy below the stable mass number of Z,
c. To understand the binding energy above the stable mass number of Z;

4) To report an attempt to fit and interrelate the SEMF binding energy coefficients.

2. Liquid Drop Model and the Semi Empirical Mass Formula

According to the well-known liquid drop model:

1) Atomic nucleus can be considered as a drop of incompressible fluid;

---

*Corresponding author.
E-mail address: seshavatharam.uvs@gmail.com, (Seshavatharam U.V.S)

Journal of Nuclear Sciences, Vol. 4, No.1, July 2017, 7-18

ISSN: 2148-7736

2) Nuclear fluid is made of protons and neutrons, which are held together by the strong nuclear force.

Mathematical analysis of the theory delivers an equation which attempts to predict the binding energy of a nucleus in terms of the numbers of protons and neutrons it contains. This equation includes five terms on its right hand side. These correspond to the cohesive binding of all the nucleons by the strong nuclear force, the electrostatic mutual repulsion of the protons, a surface energy term, an asymmetry term which is derivable from the protons and neutrons occupying independent quantum momentum states, and a pairing term which is partly derivable from the protons and neutrons occupying independent quantum spin states. The coefficients are calculated by fitting to experimentally measured masses of nuclei. Their values can vary depending on how they are fitted to the data. In the following formulae, let *A* be the total number of nucleons, *Z* the number of protons, and *N* the number of neutrons. The mass of an atomic nucleus is given by:

$$m = Zm_p + Nm_n - \left(B/c^2\right) \qquad (1)$$

where m_p and m_n represent the rest mass of a proton and a neutron, respectively, and B is the binding energy of the nucleus. The semi-empirical mass formula states that the binding energy will take the following form,

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm \frac{a_p}{\sqrt{A}} \qquad (2)$$

Here a_v *is* volume energy coefficient, a_s is the surface energy coefficient, a_c is the coulomb energy coefficient, a_a is the asymmetry energy coefficient and a_p is the pairing energy coefficient. If we consider the sum of the volume energy, surface energy, coulomb energy, asymmetry energy and pairing energy, then the picture of a nucleus as a drop of incompressible liquid roughly accounts for the observed variation of binding energy of the nucleus. As given in Table 1, the currently adopted coefficients in unit of energy (MeV) are used in semi empirical mass formula [3].

Table 1. *Adopted SEMF binding energy coefficients*

a_v MeV	a_s MeV	a_c MeV	a_a MeV	a_p MeV
15.78	18.34	0.71	23.21	12.0
15.258	16.26	0.689	22.20	10.08

3. Estimation of Stable Mass Number with Proton Number

By maximizing $B(A, Z)$ with respect to Z, it can be found the number of protons Z of the stable nucleus of atomic weight *A* as,

$$\begin{cases} Z \approx \dfrac{A}{2+\left(a_c/2a_a\right)A^{2/3}}. \\ \left(A-2Z\right) \approx \dfrac{0.4A^2}{A+200} \end{cases} \qquad (3)$$

This is roughly *A/2* for light nuclei, but for heavy nuclei there is an even better agreement with nature. By substituting the above value of *Z* back into *B*, one obtains the binding energy as a function of the atomic weight, *B(A)*. The maximization of *B(A)/A* with respect to *A* would give the nucleus which is most strongly bound or most stable. In this context, we would like to suggest that, independent of SEMF concepts, nuclear beta stability line can be understood within neutron, proton and electron rest masses. It is also possible to show that [4]:

$$\exp\left(\frac{\left(m_n-m_p\right)c^2}{m_e c^2}\right) \cong \exp\left(\frac{1.293332\text{MeV}}{0.5109989461\text{MeV}}\right) \cong 4\pi \qquad (4)$$

$$\rightarrow \left(\frac{\left(m_n-m_p\right)c^2}{m_e c^2}\right) \cong \ln\left(4\pi\right) \cong 2.53102425$$

Based on this observation and without considering the binding energy coefficients, beta stability line can be understood with the following empirical relations.

These relations can be compared with the computational relations pertaining to isotopic shift and drip lines proposed in reference [3], $N_s = 0.968051Z + 0.00658803Z^2$.

$$\left.\begin{aligned} &\text{Let, } k \cong \left(\frac{1}{4\pi}\right)^2 \cong 0.00633 \\ &A_s \cong 2Z + \left(\frac{Z}{4\pi}\right)^2 \cong 2Z + 0.00633Z^2 \cong 2Z + kZ^2 \\ &N_s \cong Z + \left(\frac{Z}{4\pi}\right)^2 \cong 2Z + 0.00633Z^2 \cong Z + kZ^2 \\ &A_s - 2Z \cong \left(\frac{Z}{4\pi}\right)^2 \cong 0.00633Z^2 \cong kZ^2 \end{aligned}\right\} \qquad (5)$$

With even-odd corrections much better correlations can be observed. For light and medium atomic nuclides, there is some mismatch, which can be attributed to shell structure and needs for further study. As indicated in

Table 2 the stable nucleon number can be estimated with its corresponding proton number by fitting.

Table 2. *The fit results for the stable mass numbers versus proton number*

Proton Number Z	**Estimated Stable Mass Number,** A_s	A_s **with Even Odd Correction**
21	44.80	45
29	63.32	63
37	82.67	83
47	107.99	107/109
53	123.79	123/125
60	142.80	142
69	168.15	167/169
79	197.52	197
83	209.62	209/211
92	237.60	238
100	263.33	262
112	303.43	302
118	324.17	324

4. Characteristic Nuclear Binding Potential and Nuclear Binding Energy Coefficients

In this analysis, it is assumed that with reference to nuclear charge radius, $R_0 \cong 1.25$ fm, and the strong coupling constant, $\alpha_s \cong 0.1186$ by the following relation.

$$B_0 \cong \left(\frac{1}{\alpha_s}\right)\frac{e^2}{8\pi\varepsilon_0 R_0} \cong 4.85655 \text{ MeV} \cong 4.86 \text{ MeV} \quad (6)$$

With this binding energy potential, nuclear binding energy can be fitted with the following two-term semi empirical relation. It should be noted that, with further research and analysis, qualitatively a simplified and unified method can be developed. With this energy potential, energy coefficients of the SEMF can be fitted in the following way.

$$\begin{cases} a_v \cong \frac{1}{2}\left(\frac{m_p}{m_e}\right)^{\frac{1}{4}} B_0 \cong 15.91 \text{ MeV}; \quad a_s \cong a_v + \frac{1}{2}B_0 \cong 18.34 \text{ MeV} \\ a_c \cong \left(\frac{m_p}{m_e}\right)^{-\frac{1}{4}} B_0 \cong 0.7425 \text{ MeV}; \quad a_a \cong a_s + B_0 \cong 23.20 \text{ MeV} \\ a_p \cong \frac{1}{2}a_a \cong \frac{1}{2}\left(a_s + B_0\right) \cong 11.6 \text{ MeV}; \\ \left(a_s - a_v\right) \cong \frac{1}{2}B_0; \quad \left(a_a - a_s\right) \cong B_0; \quad \left(a_a - a_v\right) \cong \frac{3}{2}B_0; \end{cases} \quad (7)$$

5. Proposed Method of Estimating Nuclear Binding Energy for Z=2 to 83

Starting from Z= 2 to 83,

Step 1: Close to beta stability line, nuclear binding energy can be expressed with the following semi empirical relation.

$$B_{A_s} \cong x\left[\left(\frac{A_s^2}{A_s - Z}\right) - \frac{\left(A_s - 2Z\right)^2}{A_s}\right] 4.86 \text{ MeV} \quad (8)$$

where,

$$Z \approx 2 \text{ to } 29, \; x \approx \left(\frac{Z}{30}\right)^{\frac{1}{6}} \text{ and for } 30 \ge Z \le 83, x \approx 1.0$$

$$B_A \cong xy\left[\left(\frac{A_s^2}{A_s - Z}\right) - \frac{\left(A_s - 2Z\right)^2}{A_s}\right] 4.86 \text{ MeV} \quad (9)$$

Step 2: Below and above the stable mass numbers,

where,

$$\left.\begin{matrix} Z \approx 2 \text{ to } 29, \; x \approx \left(\frac{Z}{30}\right)^{\frac{1}{6}} \\ \text{and for } 30 \ge Z \le 83, \; x \approx 1.0 \end{matrix}\right\} \left\{\begin{matrix} A < A_s, \; y \approx \left(\frac{A - Z}{A_s - Z}\right)^{\frac{2}{3}} \\ A > A_s, \; y \approx \left(\frac{A - Z}{A_s - Z}\right)^{\frac{1}{2}} \end{matrix}\right\}$$

6. Alternative Expression for Asymmetry Energy Term at Beta Stability Line

With reference to the proposed beta stability line coefficient, $k \cong \left(\frac{1}{4\pi}\right)^2 \cong 0.006333$, close to the beta stability line, with trial error, we noticed that:

For light and medium atoms,

$$\frac{\left(A_s - 2Z\right)^2}{A_s} \approx k^2 A_s N_s^{3/2} \qquad (10)$$

For heavy atoms,

$$k^2 A_s N_s^{3/2} > \frac{\left(A_s - 2Z\right)^2}{A_s} \qquad (11)$$

7. Proposed Method of Estimating Nuclear Binding Energy for Z > 83

Based on the above observation, for $Z > 83$, binding energy close to beta stability line can be expressed with the following relation.

$$B_{A_s} \cong x\left[\left(\frac{A_s^2}{A_s - Z}\right) - \left(k^2 A_s N_s^{3/2}\right)\right] 4.86 \text{ MeV} \qquad (12)$$

where $x \approx 1.0$

$$B_A \cong xy\left[\left(\frac{A_s^2}{A_s - Z}\right) - \left(k^2 A_s N_s^{3/2}\right)\right] 4.86 \text{ MeV} \qquad (13)$$

Below and above the stable mass number,

where,

$$\begin{cases} x \approx 1.0, \\ A < A_s,\ y \approx \left(\dfrac{A - Z}{A_s - Z}\right)^{\frac{2}{3}} \text{ and } A > A_s,\ y \approx \left(\dfrac{A - Z}{A_s - Z}\right)^{\frac{1}{2}} \end{cases}$$

8. Binding Energy Data Fitting

As given in Table 3 the nuclear binding energy was estimated at stable mass numbers.

Fig.1 shows nuclear binding energy at stable atomic nuclides. The plotted solid curve between the binding energy and the proton number indicates the actual or SEMF binding energy (MeV) and dotted red curve indicates estimated binding energy (MeV).

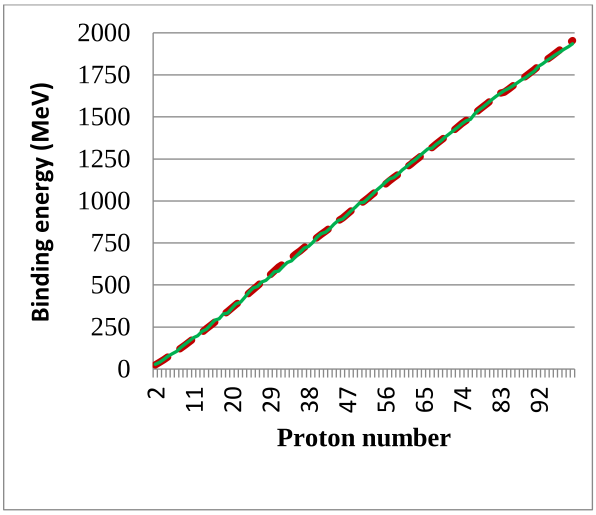

Fig. 1 *Nuclear binding energy at stable atomic nuclides*

As seen in Table 4, the isotopic binding energies are calculated for Z=20, 30, 40, 50, 60, 70, 80 and 90. The column-1 represents the proton number, column-2 represents the stable estimated mass number, column-3 represents the neutron number, column-4 represents the nucleon number, column-5 represents the binding energy calculated with SEMF, column-6 represents the binding energy calculated with proposed relations and column 7 represents the %error with respect to SEMF.

Table 3. *Estimated nuclear binding energy at stable mass numbers A_s from 4 to 263*

Proton Number Z	***Stable Mass Number*** A_s	A_s ***with Even Odd Correction***	***Estimated Binding Energy (MeV)***	***Actual [8] or *SEMF [2] Binding Energy (MeV)***
2	4	4	24.8	28.296
3	6	7	40.1	39.244
4	8	8	55.6	58.08
5	10	11	72.4	76.205
6	12	12	89.2	92.162
7	14	14	106.8	104.659
8	16	16	124.8	127.619
9	19	19	143.3	147.801
10	21	21	162.0	167.406
11	23	23	181.1	186.564
12	25	24	200.2	198.257
13	27	27	220.0	224.952
14	29	28	239.7	236.537
15	31	31	259.9	262.917
16	34	34	280.6	291.839
17	36	35	300.7	298.21
18	38	38	321.8	327.343
19	40	39	342.4	333.724
20	43	42	363.8	361.896
21	45	45	385.5	387.848
22	47	46	406.5	398.193
23	49	49	428.5	*426.34
24	52	52	450.8	456.349
25	54	55	473.2	482.075
26	56	56	494.7	492.258
27	59	59	517.5	517.313
28	61	60	539.2	526.846
29	63	63	562.2	551.385
30	66	66	585.4	578.136
31	68	67	607.5	*582.1
32	70	70	624.2	610.521
33	73	73	644.2	*634.34
34	75	74	663.0	642.891
35	78	77	683.0	*667.66
36	80	80	703.0	695.434
37	83	83	723.1	*720.46
38	85	84	741.8	728.906
39	88	87	761.8	*755.05
40	90	90	781.9	783.893
41	93	93	802.0	805.765
42	95	94	820.7	814.256

43	98	97	840.7	*836.26
44	100	100	860.9	861.928
45	103	103	881.0	884.163
46	105	104	899.6	892.82
47	108	107	919.7	915.263
48	111	110	939.8	940.646
49	113	113	960.0	963.094
50	116	116	980.1	988.684
51	118	117	998.7	*992.78
52	121	120	1018.8	1017.282
53	124	123	1039.0	*1038.77
54	126	126	1059.1	1063.909
55	129	129	1079.3	*1085.08
56	132	132	1099.5	1110.038
57	135	135	1119.7	*1131
58	137	136	1138.1	1138.792
59	140	139	1158.3	*1160.56
60	143	142	1178.5	1185.142
61	146	145	1198.7	*1203.86
62	148	148	1218.9	1225.392
63	151	151	1239.1	1244.141
64	154	154	1259.3	1266.627
65	157	157	1279.5	*1287.38
66	160	160	1299.8	1309.455
67	162	161	1318.2	*1314.32
68	165	164	1338.4	1336.447
69	168	167	1358.6	*1356.45
70	171	170	1378.8	1378.13
71	174	173	1399.0	*1397.78
72	177	176	1419.3	1418.801
73	180	179	1439.5	*1438.48
74	183	182	1459.7	1459.335
75	186	185	1479.9	1478.341
76	189	186	1498.3	1484.807
77	192	191	1520.4	1518.088
78	195	194	1540.6	1539.577
79	198	197	1560.9	1559.386
80	201	200	1581.1	1581.181
81	204	203	1601.4	1600.87
82	207	206	1621.6	1622.325
83	210	209	1641.8	1640.23
84	213	212	1646.6	*1655.76
85	216	215	1666.0	*1669.2
86	219	218	1685.3	*1685.75
87	222	221	1704.6	*1701.68
88	225	224	1723.8	*1719.13

89	228	227	1743.0	*1735.53
90	231	230	1762.1	*1753.97
91	234	233	1781.2	*1770.78
92	238	238	1803.7	1801.69
93	241	241	1822.7	*1817.31
94	244	244	1841.6	*1835.45
95	247	247	1860.5	*1851.73
96	250	250	1879.3	*1868.97
97	254	253	1898.0	*1884.38
98	257	256	1916.8	*1901.62
99	260	259	1935.4	*1917.39
100	263	262	1954.0	*1935.12

Table 4. *Estimated isotopic nuclear binding energy of Z=20 to 90*

Proton Number	***Estimated Stable Mass Number***	***Neutron Number***	***Nucleon Number***	***SEMF Binding Energy in MeV***	***Proposed Binding Energy in MeV***	***%Difference***
20	**42**	20	40	339.7	341.4	-0.50
20	**42**	21	41	350.1	352.7	-0.74
20	**42**	**22**	**42**	**363.2**	**363.8**	**-0.17**
20	**42**	23	43	371.6	372.0	-0.10
20	**42**	24	44	382.7	380.0	0.71
20	**42**	25	45	389.3	387.8	0.38
20	**42**	26	46	398.7	395.5	0.80
20	**42**	27	47	403.8	403.0	0.19
20	**42**	28	48	411.8	410.4	0.33
20	**42**	29	49	415.5	417.7	-0.53
20	**42**	30	50	422.1	424.8	-0.65
20	**42**	31	51	424.7	431.8	-1.68
20	**42**	32	52	430.2	438.8	-1.99
20	**42**	33	53	431.7	445.6	-3.21
30	**66**	30	60	509.5	518.4	-1.75
30	**66**	31	61	521.1	529.9	-1.68
30	**66**	32	62	535	541.2	-1.16
30	**66**	33	63	545.2	552.4	-1.32
30	**66**	34	64	557.8	563.5	-1.02
30	**66**	35	65	566.7	574.5	-1.38
30	**66**	**36**	**66**	**577.9**	**585.4**	**-1.30**
30	**66**	37	67	585.7	593.5	-1.33
30	**66**	38	68	595.8	601.4	-0.95
30	**66**	39	69	602.4	609.3	-1.15

30	**66**	40	70	611.5	617.1	-0.91
30	**66**	41	71	617.2	624.7	-1.22
30	**66**	42	72	625.3	632.3	-1.12
30	**66**	43	73	630.1	639.8	-1.54
30	**66**	44	74	637.2	647.2	-1.57
30	**66**	45	75	641.2	654.5	-2.07
30	**66**	46	76	647.6	661.7	-2.18
40	**90**	40	80	666.2	673.8	-1.14
40	**90**	41	81	678.6	685.0	-0.94
40	**90**	42	82	693.1	696.1	-0.43
40	**90**	43	83	704.3	707.1	-0.40
40	**90**	44	84	717.7	718.0	-0.05
40	**90**	45	85	728	728.9	-0.12
40	**90**	46	86	740.4	739.6	0.11
40	**90**	47	87	749.7	750.3	-0.08
40	**90**	48	88	761.2	760.9	0.04
40	**90**	49	89	769.6	771.4	-0.24
40	**90**	**50**	**90**	**780.2**	**781.9**	**-0.22**
40	**90**	51	91	787.8	789.7	-0.24
40	**90**	52	92	797.6	797.4	0.03
40	**90**	53	93	804.4	805.0	-0.08
40	**90**	54	94	813.4	812.6	0.10
40	**90**	55	95	819.5	820.1	-0.07
40	**90**	56	96	827.8	827.5	0.04
40	**90**	57	97	833.3	834.8	-0.18
40	**90**	58	98	840.8	842.1	-0.16
40	**90**	59	99	845.7	849.4	-0.43
50	**116**	56	106	887.6	878.4	1.03
50	**116**	57	107	898.1	888.8	1.03
50	**116**	58	108	910.4	899.2	1.23
50	**116**	59	109	920.1	909.5	1.15
50	**116**	60	110	931.8	919.8	1.29
50	**116**	61	111	940.7	930.0	1.14
50	**116**	62	112	951.6	940.1	1.21
50	**116**	63	113	960.0	950.2	1.02
50	**116**	64	114	970.2	960.2	1.03
50	**116**	65	115	977.9	970.2	0.79
50	**116**	**66**	**116**	**987.5**	**980.1**	**0.75**
50	**116**	67	117	994.6	987.5	0.71
50	**116**	68	118	1003.6	994.8	0.87
50	**116**	69	119	1010.1	1002.1	0.79
50	**116**	70	120	1018.5	1009.4	0.90
50	**116**	71	121	1024.4	1016.5	0.77

50	**116**	72	122	1032.3	1023.7	0.83
50	**116**	73	123	1037.8	1030.8	0.68
50	**116**	74	124	1045.1	1037.8	0.70
50	**116**	75	125	1050.1	1044.8	0.51
50	**116**	76	126	1056.9	1051.7	0.49
60	**142**	73	133	1099	1090.6	0.76
60	**142**	74	134	1110.2	1100.5	0.87
60	**142**	75	135	1119	1110.4	0.77
60	**142**	76	136	1129.7	1120.3	0.83
60	**142**	77	137	1138	1130.1	0.69
60	**142**	78	138	1148	1139.9	0.71
60	**142**	79	139	1155.8	1149.6	0.54
60	**142**	80	140	1165.4	1159.3	0.53
60	**142**	81	141	1172.7	1168.9	0.32
60	**142**	**82**	**142**	**1181.7**	**1178.5**	**0.27**
60	**142**	83	143	1188.5	1185.7	0.24
60	**142**	84	144	1197.1	1192.8	0.36
60	**142**	85	145	1203.5	1199.9	0.30
60	**142**	86	146	1211.6	1206.9	0.39
60	**142**	87	147	1217.5	1213.9	0.30
60	**142**	88	148	1225.2	1220.9	0.35
60	**142**	89	149	1230.7	1227.8	0.24
60	**142**	90	150	1237.9	1234.7	0.26
60	**142**	91	151	1243	1241.5	0.12
60	**142**	92	152	1249.9	1248.3	0.13
60	**142**	93	153	1254.6	1255.1	-0.04
70	**170**	91	161	1303.9	1294.8	0.70
70	**170**	92	162	1313.9	1304.2	0.73
70	**170**	93	163	1321.8	1313.7	0.61
70	**170**	94	164	1331.3	1323.1	0.62
70	**170**	95	165	1338.8	1332.4	0.47
70	**170**	96	166	1348.0	1341.8	0.46
70	**170**	97	167	1355.0	1351.1	0.29
70	**170**	98	168	1363.8	1360.4	0.25
70	**170**	99	169	1370.5	1369.6	0.07
70	**170**	**100**	**170**	**1378.8**	**1378.8**	**0.00**
70	**170**	101	171	1385.1	1385.7	-0.04
70	**170**	102	172	1393.0	1392.5	0.03
70	**170**	103	173	1399.0	1399.3	-0.02
70	**170**	104	174	1406.6	1406.1	0.04
70	**170**	105	175	1412.1	1412.8	-0.05
70	**170**	106	176	1419.4	1419.6	-0.01
70	**170**	107	177	1424.6	1426.2	-0.12

70	**170**	108	178	1431.5	1432.9	-0.10
70	**170**	109	179	1436.4	1439.5	-0.22
70	**170**	110	180	1443.0	1446.1	-0.21
70	**170**	111	181	1447.5	1452.7	-0.36
80	**200**	111	191	1509.8	1501.0	0.58
80	**200**	112	192	1518.7	1510.0	0.57
80	**200**	113	193	1525.6	1519.0	0.43
80	**200**	114	194	1534.1	1527.9	0.40
80	**200**	115	195	1540.8	1536.9	0.26
80	**200**	116	196	1549.0	1545.8	0.21
80	**200**	117	197	1555.3	1554.6	0.04
80	**200**	118	198	1563.1	1563.5	-0.02
80	**200**	119	199	1569.1	1572.3	-0.20
80	**200**	**120**	**200**	**1576.7**	**1581.1**	**-0.28**
80	**200**	121	201	1582.3	1587.7	-0.34
80	**200**	122	202	1589.6	1594.2	-0.29
80	**200**	123	203	1595.0	1600.7	-0.36
80	**200**	124	204	1601.9	1607.2	-0.33
80	**200**	125	205	1607.0	1613.7	-0.42
80	**200**	126	206	1613.6	1620.1	-0.41
80	**200**	127	207	1618.5	1626.6	-0.50
80	**200**	128	208	1624.8	1633.0	-0.50
80	**200**	129	209	1629.4	1639.3	-0.61
80	**200**	130	210	1635.4	1645.7	-0.63
80	**200**	131	211	1639.7	1652.0	-0.75
90	**230**	131	221	1699.8	1685.7	0.83
90	**230**	132	222	1707.9	1694.3	0.80
90	**230**	133	223	1714.2	1702.9	0.66
90	**230**	134	224	1722.1	1711.4	0.62
90	**230**	135	225	1728.1	1719.9	0.48
90	**230**	136	226	1735.6	1728.4	0.42
90	**230**	137	227	1741.4	1736.8	0.26
90	**230**	138	228	1748.7	1745.3	0.20
90	**230**	139	229	1754.2	1753.7	0.03
90	**230**	**140**	**230**	**1761.2**	**1762.1**	**-0.05**
90	**230**	141	231	1766.5	1768.4	-0.11
90	**230**	142	232	1773.2	1774.6	-0.08
90	**230**	143	233	1778.3	1780.9	-0.15
90	**230**	144	234	1784.8	1787.1	-0.13
90	**230**	145	235	1789.5	1793.3	-0.21
90	**230**	146	236	1795.8	1799.5	-0.20
90	**230**	147	237	1800.3	1805.6	-0.30
90	**230**	148	238	1806.3	1811.7	-0.30

90	**230**	149	239	1810.6	1817.9	-0.40
90	**230**	150	240	1816.4	1823.9	-0.42
90	**230**	151	241	1820.5	1830.0	-0.52

9. Discussion

From the above proposed relations and the estimated results in tables 2 to 4 it is possible to infer that:

1) Nucleons and electrons play a crucial role in understanding nuclear stability.

2) Strong coupling constant plays an important role in understanding the nuclear binding energy.

3) Nuclear binding energy can also be estimated with use of a single energy coefficient.

4) Z=30 seems to play an interesting role and we are working on understanding the physical significance of $\left(\frac{Z}{30}\right)^{\frac{1}{6}}$.

5) Z=53 is estimated to be stable $A_s=123$ and its estimated binding energy is 1039 MeV. Actually, it is stable at $A_s=127$. From relation (9), binding energy of ${}_{53}I^{127}$ is $\left(\frac{127-53}{123-53}\right)^{\frac{1}{2}} 1039.0 \cong 1068.27\ \mathrm{MeV}$. Actual binding energy of ${}_{53}I^{127}$ is 1072.57 MeV.

6) The authors are working in this line to deal with the nuclear binding energies and masses involving more realistic quantum physics, different shell corrections and odd-even phenomena [9-14].

10. Conclusion

Understanding and estimating nuclear binding energy with strong interaction concepts still stands as a really challenging task. So far no such model is available in physics literature. Even though some % difference is persisting in the proposed binding energy expressions, qualitatively they are very simple to follow. An interesting point to be noted is that, the number of energy coefficients can be minimized and their existence can be linked with strong interaction concepts. We are confident to say that, with further research, background physics of the proposed relations can be understood and thereby a clear and simple model can be developed.

Acknowledgements

Author Seshavatharam is indebted to professors shri M. Nagaphani Sarma, Chairman, shri K.V. Krishna Murthy, founder Chairman, Institute of Scientific Research in Vedas (I-SERVE), Hyderabad, India and Shri K.V.R.S. Murthy, former scientist IICT (CSIR), Govt. of India, Director, Research and Development, I-SERVE, for their valuable guidance and great support in developing this subject.

Conflicts of Interest

The authors have no conflict of interest.

References

[1] Von C.F. Weizsäcker, “On the theory of nuclear masses” Journal of Physics (96) 431- 458 (1935).

[2] W. D. Myers and Swiatecki. W. J. “Table of Nuclear Masses according to the Thomas-Fermi Model.” (from nsdssd.lbl.gov), (1994).

[3] P. R. Chowdhury, C. Samanta and D. N. Basu, “Modified Bethe-Weizsacker mass formula with isotonic shift and new driplines.” Modern Physics Letters A 20.21 1605-1618 (2005).

[4] C. Patrignani et al. (Particle Data Group), Chin. Phys. C (40) 100001 (2016) and 2017 update.

[5] U. V. S. Seshavatharam and S. Lakshminarayana, “Proceedings of the DAE-BRNS” Symp. on Nucl. Phys. 60 118-119 (2015).

[6] U. V. S. Seshavatharam and S. Lakshminarayana, “On the Ratio of Nuclear Binding Energy & Protons Kinetic Energy. Prespacetime Journal 6 (3) 247-255 (2015).

[7] U. V. S. Seshavatharam and S. Lakshminarayana, “Consideration on Nuclear Binding Energy Formula.” Prespacetime Journal 6 (1) 58-75 (2015).

[8] N. Ghahramany, S. Gharaati and M. Ghanaatian, “A new approach to nuclear binding energy in integrated nuclear model.” T.8, No 2 (165) 169-181 (2011).

[9] J.G. Hirsch and J. Mendoza-Temis, “Microscopic mass estimations.” J. Phys. G: Nucl. Part. Phys. 37 064029 (2010).

[10] J. G. Hirsch, C. A. Barbero, and A. E. Mariano, "Nuclear masses, deformations and shell effects." Journal of Physics: Conference Series Vol. 322 No. 1. IOP Publishing (2011).

[11] J. Duflo and A.P. Zuker, "Microscopic mass formulas. Phys." Rev. Nucl Phys C 52 R23-R27 (1995).

[12] A.E.L. Dieperink and Van P. Isacker, "Shell corrections to a liquid-drop description of nuclear masses and radii." Eur. Phys. J. A 42 269–279 (2009).

[13] Y.M. Bao, Z. He, M. Zhao and A. Arima. "Empirical formulas for nucleon separation energies." Phys. Rev. C 87 (2013).

[14] Y.M. Bao, Z. He, M. Zhao and A. Arima. "Improved Janecke mass formula." Phys. Rev C 90 (2014).