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A Study on Heavy-Ion Fusion Reactions at Extreme Subbarrier Energies

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ABSTRACT

Recent years have seen an increased interest in studying fusion reactions at extremely low subbarrier energies, although it is challenging to measure them due to their extremely small cross sections. After a brief review of the relevant literature in this area, this letter focuses on the remarkable similarities between the findings of the two effective but different models employed to analyze such reactions. The physics underlying the concealed relationship between sudden and adiabatic approaches is discussed considering the supersymmetric phase equivalent potential concept in terms of the potential functions used within their mathematical forms.

Keywords: Heavy-ion interactions, fusion hindrance, phase-equivalent supersymmetric potentials

1. Introduction

An excellent way to address the general issue of quantum tunneling in the presence of couplings, which has become a common topic in recent years in many fields of physics, is offered by heavy-ion sub-barrier fusion reactions. The Coupled-Channels (CC) treatment technique has generally been successful in analyzing nuclear reactions. However, heavy-ion fusion hindrance at extreme subbarrier energies [1], which is known as the unexpected observation of a steep falloff of fusion cross-section in this domain, has revealed that the typical CC calculations do not work at extremely low projectile energies [2]. The related differences between experimental cross sections and those of the standard CC calculations, which appeared for some colliding systems at energies well below the Coulomb barrier, led the researchers to improve coupled channel model calculations. Readers are referred to [3,4] for a more detailed investigation.

Since the low-lying collective structure of the two colliding nuclei and the subbarrier fusion dynamics are directly associated, the works in Refs. [5-7] proposed more adequate calculation techniques within the framework of CC calculations to reproduce the relevant experimental evidence. These sudden model simulations revealed that the main reason for the hindrance to fusion for reactions at extremely low energies is a thicker Coulomb barrier caused by the incompressibility of nuclear matter. Thus, the authors of [5-7] demonstrated that a successful calculation to study the hindrance problem of heavy-ion cross sections was accomplished by involving a repulsive core inside the fusion region after tunneling the Coulomb barrier. Because of the Pauli repulsion in this domain [8], which represents the interactions between the fermions of projectile and target nuclei when they are very close to one another, it is indeed necessary to include a repulsive potential that simulates the nuclear incompressibility for total overlap.

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Moving on to the adiabatic model [9,10], which has a deep potential and assumes that the two-body interaction, representing the reaction beyond the touching point, turns into a one-body potential, describing the fusion case of nuclei. The coupling effect is actually dampened in the adiabatic model after the touching. Moreover, in the sudden model fusion, hindrance arises from a cut off of high partial waves due to a shallow potential consideration while in the adiabatic model the hindrance originates from a result of the damping in the channel-coupling effects.

In the following section we glance at both model findings in analyzing different heavy ion reactions at specifically chosen energies and discuss briefly the phyiscs behind them. Section 3 presents a debate on a possible relation between the potential structures of sudden and adiabatic approaches leading to similar physical quantities. Some concluding remarks are drawn in the last section.

2. Outcomes of Sudden and Adiabatic Models

Within the framework of the sudden model, one usually considers a shallow and thick potential barrier. The repulsive phenomenological core, in this model approach, due to the saturation property of nuclear matter is taken into account in the inner region of the barrier while the outer region of the potential is constructed with a double folding procedure (Michigan-three range-Yukawa or in brief M3Y). The basic assumption, in this case, is that the reaction occurs very fast and that the density in the area where the interacting nuclei overlap is doubled. It was shown that [5-7] the improved CC calculations with such a shallow potential reproduce the steep fall-off phenomenon for heavy ion interactions accurately. For instance, to illustrate the shape of this potential, as shown in Fig.1, Mişicu and Esbensen [5] compared the spherical heavy-ion potential, M3Y+repulsive, that is shallow, for the system of ^{64}Ni+^{64}Ni to deep potentials having a different structure that has previously been used. The differences in the thickness and depth of these three potential behaviors are obvious inside the region where the overlapping occurs.

Potentials such as the Akyüz-Winther (A-W) and Proximity-77 in the figure above yield physically acceptable barriers but the fact that they are not able to reproduce the data far below the barrier. This is indeed an indication that the ion-ion potential has an alternative form in the inner part of the barrier like, M3Y+repulsive one.

Along this line, from Fig.2, it is clear that the best analysis of the related fusion reaction is given by the

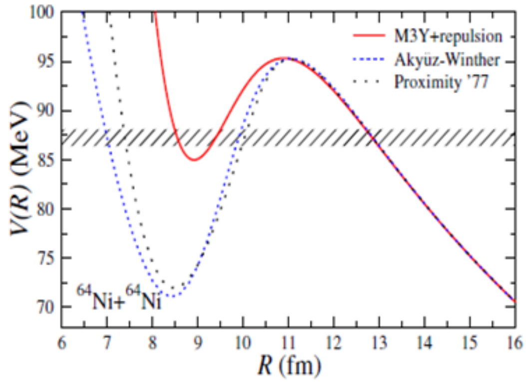

Fig.1 *Various spherical ion-ion potentials for ^{64}Ni + ^{64}Ni. Taken from [5].*

improved CC model with M3Y+repulsive short-range nucleon-nucleon potential, which agrees with the observed data across the entire range of center of mass energies under concern. Akyüz-Winther potential and the other simple CC theoretical calculation without inclusion of coupling (NOC), have no capability to reproduce the corresponding cross-section data for the overall domain.

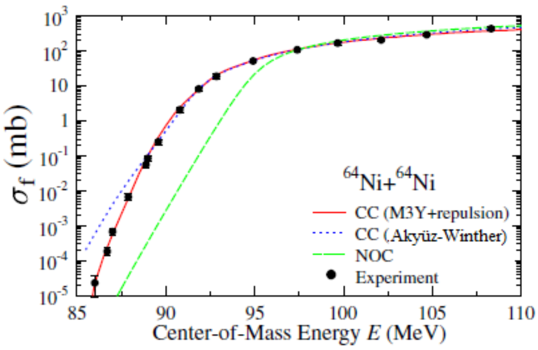

Fig.2 *Experimental fusion cross section for the system ^{64}Ni +^{64}Nis compared to various calculations. Quoted from Ref. [5].*

To identify explicitly the various circumstances in which the hindrance through the subbarrier fusion occurs, the authors [6] proposed the use of adding two more physical quantities: one is the astrophysical S factor, $S = E\sigma(E)\exp(2\pi\eta)$, where E is the center of mass energy, $\sigma(E)$ is the corresponding fusion cross section, $\eta = Z_pZ_te^2/(\hbar\upsilon_{rel})$ is the Sommerfeld parameter, being with Z_p and Z_t are the nuclear charges of projectile and target nuclei respectively, and finally υ_{rel} is the beam velocity. The other one is the logarithmic derivative $(E) = d\frac{[\ln(\sigma E)]}{dE}$. This kind of search on such quantities serves as a testing ground of the model calculations, one of which is given below. In Fig.3, the researchers in [6] compared the experimental S factors for different systems with those calculated with the M3Y+repulsion and the A-W potentials.

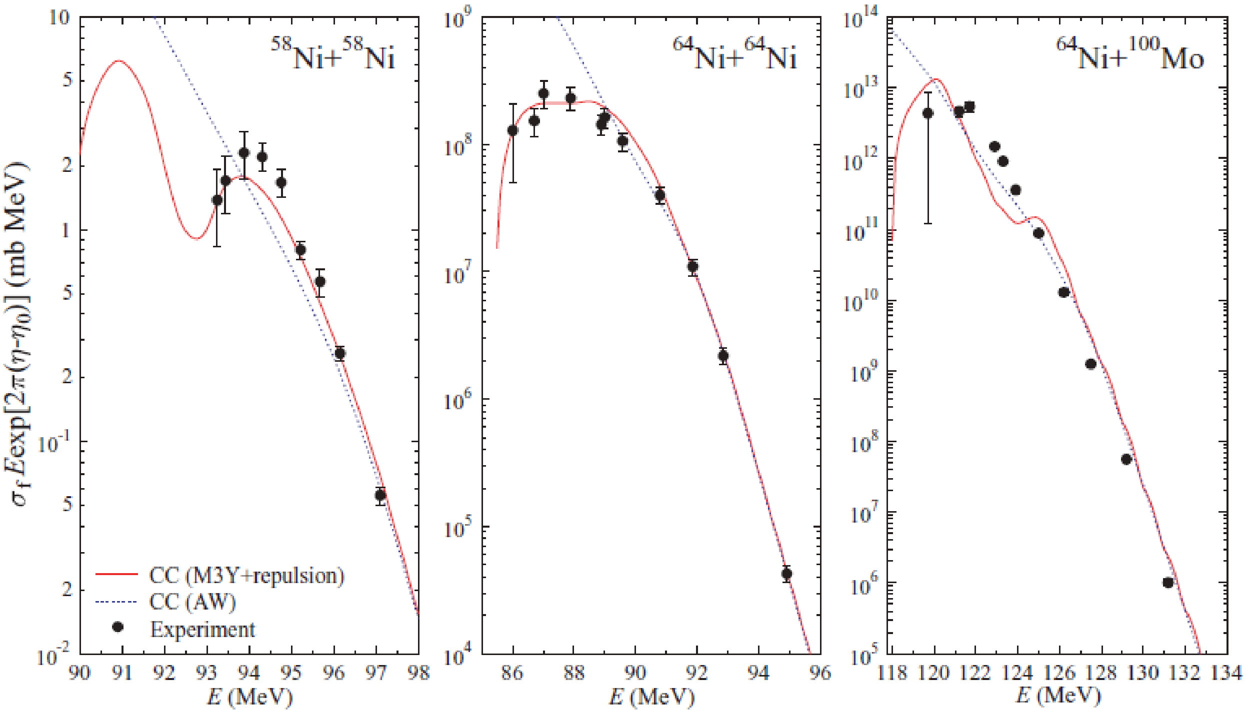

Fig.3 *Experimental S factors for the various systems compared with the refined CC calculations performed with the M3Y+repulsion and A-W potentials. Taken from [6].*

The one with repulsion having shallow nature provides a good description of the available data and, most importantly, reproduces the experimental *S* factor's trend to bring about a maximum. The calculations performed with M3Y plus repulsive piece also predict maxima at lower energies, supporting the accuracy of the improved CC calculations within the frame of the sudden model.

The sudden approximation, however, tends to overestimate the tunneling probability at energies much below the barrier if the internal system's excitation energies are high. One must think about the problem in this case at the adiabatic limit. Adding a damping factor is the most significant amendment to the standard CC treatment within the context of the adiabatic approach. The fusion cross sections, *S* factors and logarithmic derivatives calculated within this adiabatic model using the Yukawa-plus-exponential (YPE) potential are in very good agreement with experimental data [10] for a variety of systems. The authors pointed out that, except for medium-light mass systems, the energy at the touching point does correlate with the threshold energy observed in case of hindering phenomena in various considerations. See, for instance, [9-10] for additional information on the adiabatic model calculations.

The difference between the sudden and the adiabatic models is schematically illustrated in Fig. (4).

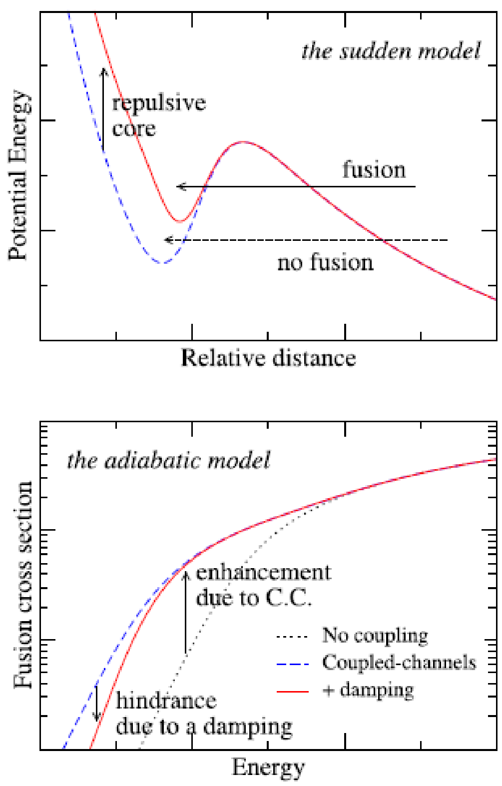

Fig.4 *Schematic illustration of the difference between the sudden model and the adiabatic model for deep-subbarrier fusion hindrance. Quoted from [3].*

Fig.5 also shows the findings of ensuing analyses [10] performed within the sudden and adiabatic models. The results for the cross-section calculations in these two models are almost the same. For an explicit judgment on the reliability of the model findings, one would require measurements at lower incident energies, where the predictions regarding *S* factors deviate from each other only in this region. Such a measurement is fascinating, though it would be very challenging.

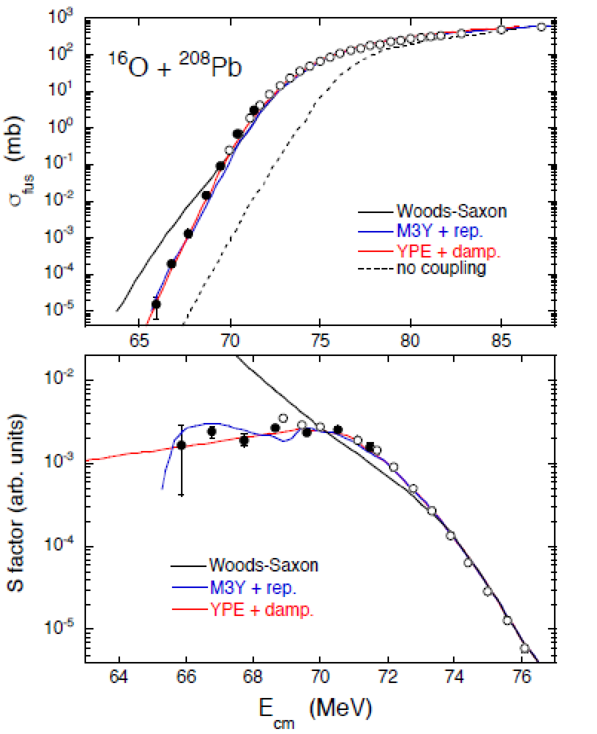

Fig.5 *Fusion cross sections (upper panel) and astrophysical S factor (lower panel) of ^{16}O + ^{208}Pb compared to several CC calculations. Taken from [10].*

3. Discussion on the possible relation between the Adiabatic and Sudden approaches

While the adiabatic model takes into account a deep potential consideration, the sudden model uses unlikely a shallow internuclear potential. Fig.6 illustrates schematically the fusion dynamics with a shallow nucleus-nucleus potential (sudden model) and a deep internucleus potential (adiabatic model).

Employing a local potential is a reasonable way to explain the interaction between two composite particles. The bound states created by the interacting particles and their scattering characteristics are frequently reproducible by such a potential. However, due to the internal structure of these particles, there may be some confusion between two families of potential states: shallow potentials, which express physical bound states, and deep potentials, which have in addition non-physical bound states that simulate the Pauli principle's effect on the constituent fermions [11,12].

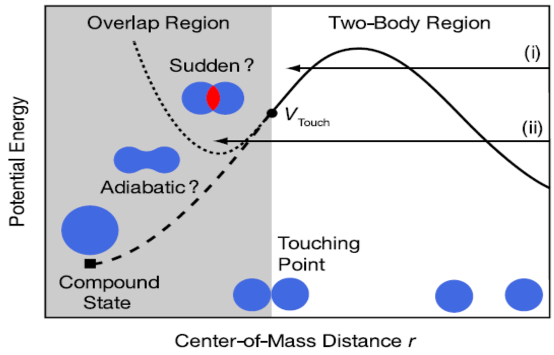

Fig.6 *A schematic illustration of an internucleus potential and the dynamics of fusion reactions in the sudden and the adiabatic models. Quoted from Ref. [10].*

In the single-channel case, supersymmetric quantum mechanics [11] provides an effective clear prescription to remove smoothly non-physical bound states from a given deep potential. In coupled-channel cases, which the sudden and adiabatic models use to analyze the heavy-ion fusion interactions, there is an ambiguity between deep and shallow potentials. The derivation of phase-equivalent potentials (PEP), which is an ideal tool for this purpose, was successfully extended [12] to coupled channels within the framework of supersymmetric quantum mechanics in order to address such ambiguities. The authors of [12] clearly showed that by removing the deep Pauli forbidden bound states, a shallow effective phase equivalent potential can be obtained, which has a repulsive core like existing physically acceptable shallow potentials in the related literature. The wave function of the deep potential has a node, as well discussed in Refs. [12] and [13]. This node disappears in the wave function of the shallow potential and is replaced by an r^3 behaviour nearby the origin, in agreement with the singularity modification. In case of an original deep potential, identical fermions occupying the same states correspond to the lowest levels at small distances like overlapping region. By forcing the particles to occupy higher-lying orbits and providing the required repulsion, namely removing these non-physical bound states avoids violation of the Pauli principle. It was made clear by the research in [12] and [13] that both wave functions exhibit extremely similar asymptotic behavior. This supports the phase equivalence of the two potentials and is also valid for scattering states [11]. In conclusion, the supersymmetric quantum mechanical framework enables the construction of PEP [12] in the coupled channel as it does in single-channel cases.

Apart from this, the Pauli principle can also be interpreted in terms of Pauli attraction rather than Pauli repulsion, although that it is generally believed that it results in a repulsive core, as was the case in the sudden model [5,6]. Notice that Pauli principle aims to quench the radial wave function at short distances and that the Pauli repulsion and the Pauli attraction both accomplish this similarly. Ohkubo [14] has recently put forwarded this point for a nucleon-nucleon interaction, supporting the results in [11–13]. The corresponding Pauli attraction naturally implies that the potential must be deep enough to hold the prohibited states, as in the case of supersymmetric partner potentials, as the physically relevant wave function must be orthogonalized to Pauli forbidden states. This point makes a link between the potentials used by sudden and adiabatic potentials.

This short discussion reveals a possibility regarding a hidden supersymmetric relation between the sudden and adiabatic model analysis of fusion observables for heavy ion interactions. Considering the work in [13], which investigates wave function-sensitive properties of the supersymmetric potentials by a halo transfer reaction, it is helpful to remind that a deep potential and its phase equivalent shallow partner, that are used for building the related entrance and exit channel wave functions for the given fusion process, are constructed with identical phase shifts so that any differences in physical quantities, such as relevant cross-section analyses in reactions, can only be explained by differences in the corresponding wave functions of the partner potentials. Along this line, the researchers in [13] observed that reconstructed PEP have led to relative motion wave functions very similar to those generated by the deep potentials outside the core region, but with no radial node at small distance due to the singularity of the shallow partner potential. As a result, it is anticipated that both types of potentials will exhibit somehow different off-shell behaviours and produce results that are qualitatively different. However, interestingly, no major difference was found between the *rms* radii calculated from these quite different two-body interactions. Considering transfer reaction $^{11}Be(p,d)^{10}Be$, a follow-up study in Ref. [13] on the effects of utilizing such phase equivalent two-body potentials to characterize weakly bound ^{11}Be and deuteron nuclei in three-body model calculations, as entrance and exit channel wavefunction compononents, yielded curiously similar results. Hence, the authors of [13] concluded that the short-range behaviour of the corresponding wave functions for the deep and phase equivalent shallow potentials, which coincide at large distances but differ at small distances by the additional node appearing inside the core, is not significant for the analysis of such transfer reaction observables.

Given the above-discussed research findings, and with the expertise gained from the work in [13], we thus propose that, despite the structural differences between the potential functions used in these analytical treatment techniques, there may be a similar relationship between the potentials used by sudden and adiabatic models. This could be the reason for the remarkable similarities between the analyses of fusion observables for heavy-ion systems seen in Fig. 5.

To justify our entire debate here, one may also consider the mean angular momentum calculations of the compound nucleus, illustrated in Fig.7, performed by sudden and adiabatic models.

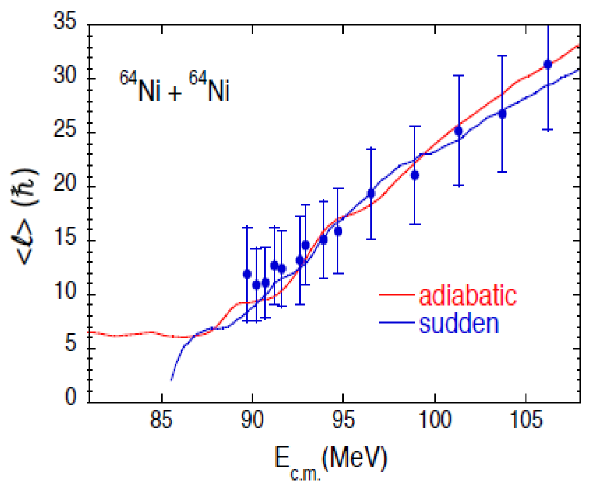

Fig.7 *Average angular momentum of compound nucleus vs. incident energy for ^{64}Ni + ^{64}Ni. Taken from [15]. The results of the sudden model were performed using the M3Y+ repulsive potential.*

This physical quantity is evaluated in a way [16] that

$$\prec \ell \succ (E) = \frac{\frac{\pi}{k^2}\sum_{\ell} \ell\,(2\ell+1)T_{\ell}}{\frac{\pi}{k^2}\sum_{\ell}(2\ell+1)T_{\ell}} \qquad (1)$$

in which $\sigma_\ell(E) = (\pi/k^2)(2\ell+1)T_\ell(E)$ is the partial wave cross section leading to the total excitation function, $\sigma_{fus}(E) = \sum_\ell \sigma_\ell(E)$, and T_ℓ is the quantum-mechanical transmission probability through the potential barrier. The radial wave function $u_\ell(r)$ in the exit channel for the partial wave ℓ is in relation with T_ℓ as given by [3,4,16]

$$u_\ell(r) = \sqrt{\frac{k}{k_\ell(r)}}T_\ell \exp\left(-i\int_r^{r_{abs}} k_\ell(r')\,dr'\right) \quad (r \le r_{abs}) \qquad (2)$$

where the integral region defines inside of the Coulomb barrier and $k_\ell(r)$ is the local wave number

$$k_\ell(r) = \sqrt{\frac{2\mu}{\hbar^2}\left(E - U(r)\right)} \quad ,$$
$$U(r) = V_N(r) + V_C(r) + \frac{\ell(\ell+1)\hbar^2}{2\mu r^2} \qquad (3)$$

which takes into account the real part of the full internuclear potential, $U(r)$, and r_{abs} is the absorption radius, together with k being the incident wave number ($= 2\mu E/\hbar^2$). Obviously, μ and E are the reduced mass and the incident energy in the center of mass frame for the reaction of interest. With this consideration, it is clearly seen that the effects of the related terms T_ℓ , $U(r)$ and $u(r)$ in Eqs. (2) and (3) are not remarkably large in the calculated physical observables, as shown in Figs. 5 and 7, justifying that the difference in potential and wavefunction behavior that appeared in these treatments does not cause a significant distinction in the theoretical reproduction of physical quantities at low energies similar to the work in [13]. This point once more reveals a prospective interconnection between the sudden and adiabatic models in terms of their potential descriptions used for analysing fusion reactions at low energies.

For a closing remark, we draw the attention of the reader to the most recently published [17] an impressived work in which the curvature of the potential barrier,$\hbar w$, has been modified as $\hbar w \rightarrow (\hbar w)\exp\left[\lambda\frac{E-V}{V}\right]$ and shown that the modified Wong formula with this novel term reproduces fusion crosss sections quite well for various systems, involving heavy ion interactions, across the whole energy range including fusion hindrance phenomenon, unlike the original expression. Considering the analysis in [17] one can clearly see that the deficiency, being the insensitivity of barrier properties such as radius (R), height (V) and curvature $(\hbar w)$ to the angular momentum, in the original Wong formula has been removed through the modification of only the curvature term. The work in [17] thus demonstrates that $\ell-$dependence of R, V and $\hbar w$ may be correlated and can be simulated with a single $\ell-$dependence of the $\hbar w$ term. At this stage, remembering also the $\ell-$dependency of a shallow phase equivalent partner potential due to elimination of unphysical Pauli forbidden state(s), it is not hard to establish a connection to the shallow nature of the potential behaviour in the sudden model because of the same reason: Pauli repulsion between the fermions of reacting nuclei inside the barrier. This plausible connection seems another evidence for justifying the whole discussion in this section.

4. Concluding Remarks

Although the physical recipe behind the hindrance is given differently through the sudden and the adiabatic treatment technique, both emphasize the importance of dynamical effects within overlapping regions. Overall, it is not explicitly accepted that the hindrance phenomenon is better described with a two-body potential producing a shallow potential pocket, or if an adiabatic approach is more appropriate due to its different potential structure consideration. Discriminating between these two models will require challenging measurements. We believe that this simple and intuitive discussion through this paper would shed light on the related area.

Conflict of Interest

The authors have no conflict of interest.

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