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# Journal of Nuclear Sciences

p-ISSN: 2147-7736, e-ISSN:2148-3981

Journal homepage: <http://jns.ankara.edu.tr>

DOI: 10.1501/nuclear.2023.59

## Excited Energy Spectra for He and Be Isotopes from Different Shell Model Interactions

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Received 06.12.2022; received in revised form 08.02.2023; accepted 09.02.2023

### ABSTRACT

The nuclear shell model is one of the common methods, used to investigate the nuclear structures of atomic nuclei. In this study, this model was used to investigate the nuclear structures of He and Be isotopes.  ${}^4\text{He}$  nucleus has been taken as a closed-shell core nucleus. Considering  $p_{3/2}$  and  $p_{1/2}$  as single-particle orbits in model space, different two-body interactions were used between valence nucleons. The best results from the interactions were compared with each other and the available experimental values.

**Keywords:** Nuclear shell-model, He, Be, energy level, spin-parity

### 1. Introduction

In the orbital model of the atom, electrons are thought to be in the orbits which are thought to exist around the atomic nucleus. This arrangement of electrons occurs according to the Pauli exclusion principle, and two electrons with the same quantum number can never be in the same orbital. Each orbital has a maximum number of electrons it can hold in relation to its quantum numbers. As a result of filling the orbitals with electrons in this way, it is known that atoms with a certain number of electrons are more stable than others. These atoms are known as noble gases. It has been seen that a model like this one can also be applied to protons and neutrons, whose common names are nucleons, located in the nucleus of the atom. In this model, which is called the nuclear shell model [1-5], nucleons are placed separately in orbits within the nucleus according to the Pauli principle. Like the noble gases, some nuclei with nucleon numbers have been observed to be more stable than others, which are called magic numbers (2, 8, 20, 28, 50,

82, and 128) in nuclear physics [6, 7]. Double magic nuclei, which have magic numbers of both neutrons and protons, are spherical and very stable. Single particle trajectories with a magic number have too much distance between the subsequent trajectories, causing the trajectories to be found in groups. These groups are called shells, from which the name of the nuclear shell model comes. These shells are named according to the orbits they contain. Recent theoretical and experimental studies indicate that there may be new magic numbers different from the existing ones for some shells, or the existing ones may not be magic [8].

In the nuclear shell model calculations, even magic number nuclei are considered as core nuclei, and valence nucleons greater than this are included in the calculations. It is assumed that the nucleons that give  $J=0$  total angular momentum in the core do not move. Accordingly, it is not possible for these nucleons to be

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included in the valence nucleons by leaving the essence. It is assumed that the valence nucleons may be dispersed in the shell just above the core. This shell is treated as the model space. Nucleons in the model space can be in any combination in each orbital. Different placements result in different energy levels of the nucleus. The increase in the orbital capacities of the orbits in the model space and the increase in the number of valence nucleons make the calculations very difficult. In this study, the nuclear properties of the He and Be nuclei in the p-shell were investigated by using the  $^4\text{He}$  core. The isotopes studied have either or both non-core protons and neutrons in the model space. Protons and neutrons were dispersed separately in each combination in the p-shell model space. Low-lying excited level spin-parity and energy values of  $^5\text{-}^9\text{He}$  and  $^7\text{-}^{12}\text{Be}$  isotopes were calculated. For the calculations, the Kshell shell-model computer code was used [9, 10].

## 2. Material and Methods

The nuclear shell model is the most appropriate tool to describe the low-energy structure of the atomic nucleus [11]. In this model, nucleons are assumed to move in an independent central potential well. After the strong spin-orbit interaction was proved to be an important component, single particle orbital arrays and magic numbers (2, 8, 20, 28, 50, 82, and 126) were finalized with the inclusion of spin-orbit interaction [12, 13]. As it is known, nuclei with magic numbers in terms of both protons and neutrons are more stable and spherical than the others. Although the shell model of the nucleus is like the shell model of the atom, it also contains many difficulties. The first of these difficulties is the narrowing of the working area since the dimensions of the atom are on the order of  $10^{-10}$  m and that of the nucleus  $10^{-15}$  m. Another difficulty is the coexistence of two different types of particles (protons and neutrons) in the nucleus, instead of electrons, which are the only type of particle as in the atom. In the shell model of the atom, electrons move independently in a central potential well in the orbitals of the atom. The central potential is due to the positive charge of the nucleus and the average repulsive interaction of the electrons. However, there is no such central potential in the shell model of the core. Also, in this model, there is no clear definition between nucleons, as in the interaction of electrons with each other or the nucleus defined by the Coulomb interaction.

Calculating nuclear energy levels is a very difficult task. The main reason for the difficulty is that the nature of the interaction between free protons and neutrons, in other words the strong nuclear interaction, is not well known. If we consider a nucleus with a few valence nucleons outside the closed shells, the energies of the levels can be divided into three parts. The first part is the binding energy of the closed shells ( $^4\text{He}$  in this study), the second one is the sum of the kinetic energies of the valence nucleons and the single nucleon energies that include

their interactions with the nucleons of the core. The third one is the interaction of valence nucleons with each other. Among these, calculating the binding energies of closed shells is the most difficult one. The easiest one is the calculation of the interaction between the valence nucleons. If these valence nucleons are in a single orbital, it is sufficient to know only the matrix elements of the effective interaction between the nucleons in that orbital. If the valence nucleons are distributed over several orbitals, the differences between the single nucleon energies (single-particle energies) are also needed, which can often be taken from the experimental data. One of the most important points in shell model calculations is the selection of the effective interaction to be used between the valence nucleons [14].

Due to the difficulty of not knowing the individual interactions between nucleons, an average potential generated by other nucleons (mean field approximation) is involved instead of these interactions. Thus, the problem considered in the nuclear shell model, the many-body problem that considers all nucleons in the nucleus, is reduced to a few-body problem that takes into account only the valence nucleons.

In the operations performed in the matrix formalism for the multi-particle systems, the dimensions of the Hamiltonian matrix increase to very high orders (1010) as the size of the model space and the number of nucleons increase. To obtain the eigenvalues, the matrices are diagonalized using appropriate algorithms such as Lanczos, and the solution is reached. For this purpose, there are many computer codes developed to make nuclear shell model calculations in the literature. Examples of these are given by [9, 15-19]. In the calculations performed in this study, the Kshell code was used. Running on the Linux operating system, this code allows performing nuclear shell model calculations with M-scheme representation using the Lanczos method.

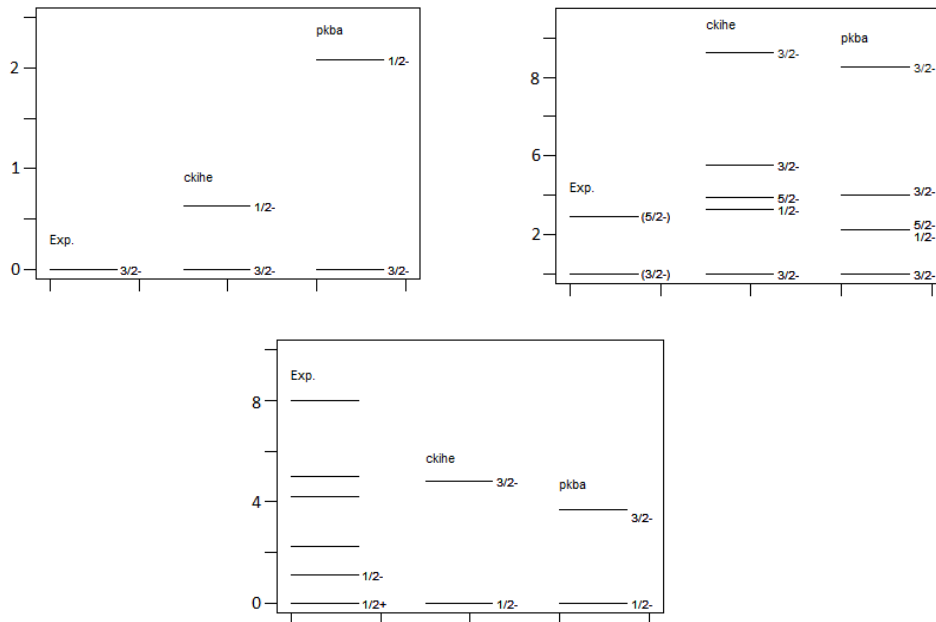
## 3. Results and Discussion

In our study, we considered the p-shell as the model space. This space consisted of  $p_{3/2}$ , and  $p_{1/2}$  orbits located on the  $^4\text{He}$  core. The core nucleus had 2 protons and 2 neutrons. Since the number of neutrons of the He nucleus to be studied varied from 3 to 7, the numbers of valence neutrons in the model space were from 1 to 5 for  $^5\text{He}$  to  $^9\text{He}$  isotopes. For the Be isotopes under investigation, the number of protons was 4 which corresponds to 2 valence protons in the model space. The neutron number varied from 3 to 8 in the Be isotopes of  $^7\text{Be}$  to  $^{12}\text{Be}$ , the number of valence neutrons being between 2 to 6 for the isotopes. In the shell model calculations performed for He and Be isotopes, different two-body interaction matrix elements available in the literature have been used. These were cki, ckihe, ckii, ckiiipn, ckipn, ckpitpn, ckpivpn, ckpot, ckppn, mp, msdisa, pcospn, pwt, pjp, pjppn, pj, pjtpn, pkba, pkuo, pkupn, pmom, pmompn, pw, pwbp, pwbt, pwt, su3p and su3ppn. Among them, the results of the two that

gave better results for He isotopes were presented in this study. These were ckihe and pkba two-body interactions. The single-particle energies for  $p_{3/2}$  and  $p_{1/2}$  orbits are 4.932 and 0.992 MeV for ckihe, 3.828 and 1.744 MeV for pkba respectively. The number of two-body matrix elements was 15 for these two different interactions. For Be isotopes, the two interactions that gave better results were mp and pkupn. The numbers of two-body matrix elements were 15 and 44 for mp and pkupn, respectively. The single-particle energies were 1.2409 and 1.2735 MeV for mp, 3.990 and 0.000 MeV for pkupn.

The energy spectra of the results of the calculation for the odd-numbered He isotopes are given in Fig.1 in comparison with the available experimental data. For  $^5\text{He}$ , only the ground-state spin and parity were available as  $3/2^-$  as experimental data in the literature. In the calculations performed with both interactions within the scope of this study, the ground state spin and parity were obtained correctly. With the ckihe interaction, the first excited state spin and parity were calculated as  $1/2^-$  and the energy value of this level was obtained as 0.633 MeV. In addition, the pkba interaction gave the same spin and parity for the first excited state, while giving an energy value of 2.084 MeV. The experimental ground-state spin and parity for the  $^7\text{He}$  isotope was  $3/2^-$ , and there was an ambiguity in this value.

$3/2^-$ . The spin-parity and energy values of the experimental first excited level were  $5/2^-$  and 2.920 MeV. The energy of the  $5/2^-$  level from ckihe and pkba were calculated as 3.866 and 2.248 MeV, respectively. Moreover, the  $1/2^-$  level was inserted between the ground state, and this level in the calculations was performed with both interactions. The energy of this level was calculated as 3.256 and 2.216 MeV for ckihe and pkba, respectively. In the literature, the next level of energy is 5.800 MeV, and the spin-parity value was uncertain. According to the results obtained from the ckihe interaction, it was seen that there were two  $3/2^-$  levels after the  $5/2^-$  level. The energies of these levels were 5.510 and 11.238 MeV, respectively. According to the pkba interaction, two  $3/2^-$  levels came after the  $5/2^-$  level and their energies are 3.976 and 10.511 MeV. For  $^9\text{He}$ , there were only two levels in the literature for which spin-parity was known. The ground state was  $1/2^+$  and the first excited state is  $1/2^-$  with an energy of 1.100 MeV. The spin-parity of the next levels is uncertain, and their energies were 2.260, 4.200, 5.000 and 8.000 MeV. In the calculations, the ground state was calculated as  $1/2^-$  according to both interactions. The first excited state was obtained as  $3/2^-$ . The energy of the first excited state for ckihe and pkba was 4.796 and 3.701 MeV, respectively. The pkba interaction was more in line with the available experimental data for  $^7\text{He}$ .



**Fig.1** The energy spectra for  $^5\text{He}$  (top-left),  $^7\text{He}$  (top-right) and  $^9\text{He}$  (bottom) isotopes from literature experimental data and the results from shell-model calculations with ckihe and pkba interactions

In the calculations performed, it was confirmed by both interactions that the ground state spin and parity were

The results of the shell model calculations for He isotopes with even mass numbers are presented in Fig.2,

together with the experimental values. The ground-state spin-parity value was calculated with both interactions in agreement with the experimental values. The spin-parity and energy values of the first excited state were experimentally  $2^+$  and 1.797 MeV. These values were obtained as  $2^+$  and 1.894 MeV with the ckihe interaction, and as  $2^+$  and 1.749 MeV with the pkba interaction. The experimental energy value of the next level was 5.600 MeV, and spin and parity were ambiguous. It has been suggested that the spin-parity of this level can be  $2^+$ ,  $1^-$  or  $0^+$ . The energy of the next level was 13.900 MeV and its

spin-parity is  $1^-$  or  $2^-$ . When the experimental spectrum was examined, it was seen that there were 15.500 and 24.200 MeV levels without spin-parity values after these levels. In the calculations, it was seen that  $1^+$  6.124 MeV,  $2^+$  7.268 MeV and  $0^+$  12.467 MeV levels emerged with ckihe.  $2^+$  5.727 MeV,  $1^+$  6.563 MeV and  $0^+$  11.572 MeV levels were obtained from pkba interaction. For these isotopes, the pkba interaction was more in agreement with the available experimental data.

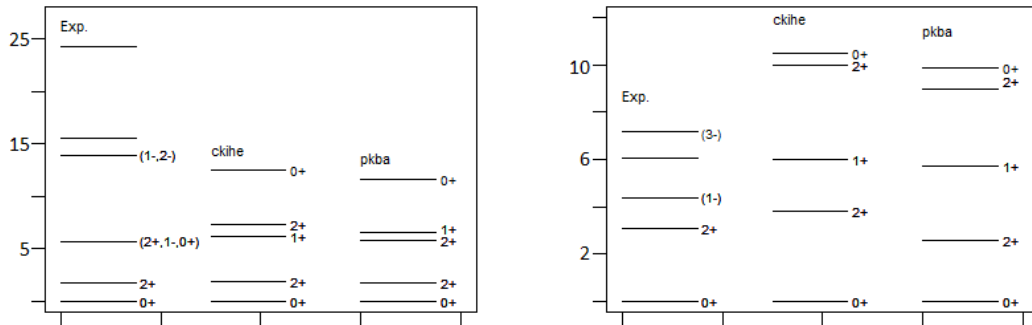


Fig.2 The energy spectra for  ${}^6\text{He}$  (left) and  ${}^8\text{He}$  (right) isotopes from literature experimental data and the results from shell-model calculations with ckihe and pkba interactions

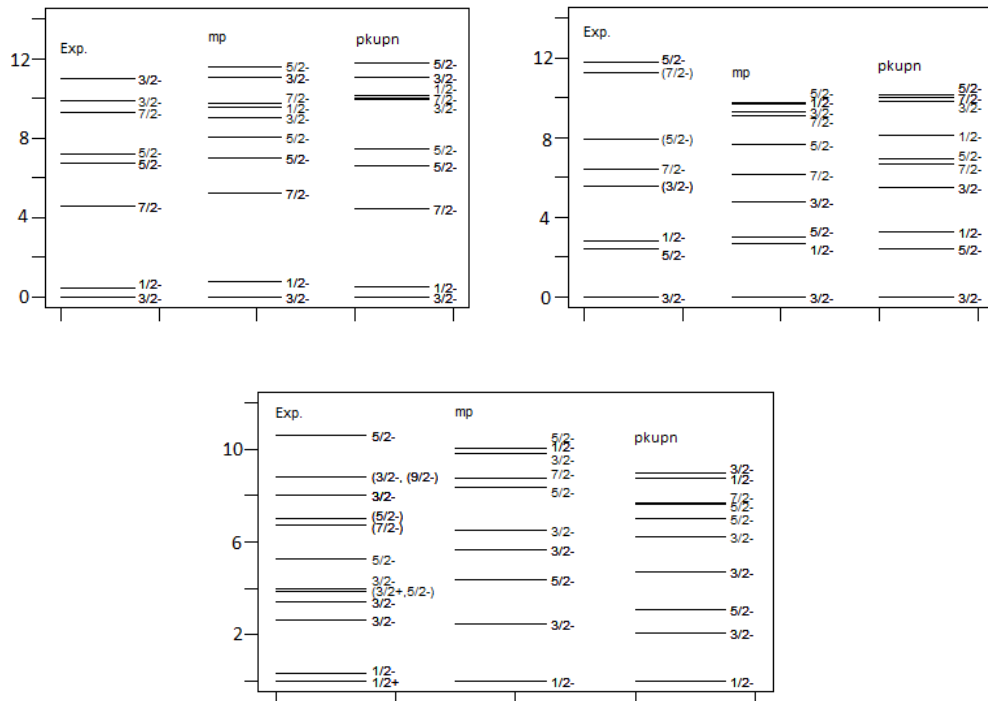


Fig.3 The energy spectra for  ${}^7\text{Be}$  (top-left),  ${}^9\text{Be}$  (top-right) and  ${}^{11}\text{Be}$  (bottom) isotopes from literature experimental data and the results from shell-model calculations with mp and pkupn interactions

In the calculations performed for  $^7\text{Be}$ , the ground-state spin-parity value was obtained as  $3/2^-$  in accordance with the experimental value with both interactions. The experimental values of the first, second, third, and fourth excited states were  $1/2^-$  0.429 MeV,  $7/2^-$  4.570 MeV,  $5/2^-$  6.730 MeV and  $5/2^-$  7.210 MeV. The values obtained by the mp interaction were consistent with the experimental data, and these values were  $1/2^-$  0.768 MeV,  $7/2^-$  5.260 MeV,  $5/2^-$  7.022 MeV, and  $5/2^-$  8.047 MeV. From the calculations made with pkupn, it was seen that this sequence was  $1/2^-$  0.537 MeV,  $7/2^-$  4.438 MeV,  $5/2^-$  6.584 MeV and  $5/2^-$  7.467 MeV. After these levels, the experimental energy spectrum was  $7/2^-$  9.270 MeV,  $3/2^-$  9.900 MeV,  $3/2^-$  11.010 MeV and  $1/2^-$  17.000 MeV. However, according to the results obtained from the mp interaction, there was a sequence of  $3/2^-$  9.044 MeV,  $1/2^-$  9.581 MeV,  $7/2^-$  9.779 MeV,  $3/2^-$  11.096 MeV and  $5/2^-$  11.568 MeV. The results of the pkupn interaction, in which  $7/2^-$  and  $1/2^-$  levels were observed to be displaced, were  $3/2^-$  9.933 MeV,  $7/2^-$  10.044 MeV,  $1/2^-$  10.148 MeV,  $3/2^-$  11.064 MeV and  $5/2^-$  11.754 MeV.

In the calculations for  $^9\text{Be}$ , the ground-state spin-parity value was obtained as  $3/2^-$ , which was consistent with the experimental data. Since only negative parity cases were calculated, comparisons were made on negative parity cases. When the experimental spectrum was examined, the negative parity states above the ground state were  $5/2^-$  2.429 MeV,  $1/2^-$  2.780 MeV,  $(3/2^-)$  5.590 MeV,  $7/2^-$  6.380 MeV,  $(5/2^-)$  7.940,  $(7/2^-)$  11.282 MeV and  $5/2^-$  11.810 MeV. According to the results of the calculations made with the mp interaction, this sequence was  $1/2^-$  2.697 MeV,  $5/2^-$  3.014 MeV,  $3/2^-$  4.802 MeV,  $7/2^-$  6.133 MeV,  $5/2^-$  7.630 MeV,  $7/2^-$  9.114 MeV  $3/2^-$  9.321 MeV,  $1/2^-$  9.691 MeV and  $5/2^-$  9.752 MeV. When the pkupn interaction results were examined, it was seen that  $5/2^-$  2.429 MeV,  $1/2^-$  3.250 MeV,  $3/2^-$  5.488 MeV,  $7/2^-$  6.654 MeV,  $5/2^-$  6.910 MeV,  $1/2^-$  8.096 MeV,  $3/2^-$  9.842 MeV,  $7/2^-$  10.008 MeV and  $5/2^-$  10.131 MeV.

According to the shell model calculations for  $^{11}\text{Be}$ , the ground-state spin-parity was obtained as  $1/2^-$ . On the other hand, it was seen that the experimental data is  $1/2^+$ . Experimentally excited states were  $1/2^-$  0.320 MeV,  $3/2^-$  2.654 MeV,  $3/2^-$  3.400 MeV,  $(3/2^+, 5/2^-)$  3.889 MeV,  $3/2^-$  3.955 MeV,  $5/2^-$  5.255 MeV,  $(7/2^-)$  6.705 MeV,  $(5/2^-)$  7.030 MeV,  $3/2^-$  8.020 MeV,  $3/2^-$  8.200 MeV,  $3/2^-$  or  $(9/2^-)$  8.813 MeV and  $5/2^-$  10.590 MeV. Excited states according to the results obtained from mp interaction have been calculated as  $3/2^-$  2.445 MeV,  $5/2^-$  4.343 MeV,  $3/2^-$  5.688 MeV,  $3/2^-$  6.483 MeV,  $5/2^-$  8.382 MeV,  $7/2^-$  8.775 MeV,  $3/2^-$  9.821 MeV,  $1/2^-$  9.840 MeV and  $5/2^-$  10.066 MeV. With the pkupn interaction, these states were  $3/2^-$  2.042 MeV,  $5/2^-$  3.104 MeV,  $3/2^-$  4.686 MeV,  $3/2^-$  6.208 MeV,  $5/2^-$  6.997 MeV,  $5/2^-$  7.647 MeV,  $7/2^-$  7.660 MeV,  $1/2^-$  8.738 MeV and  $3/2^-$  8.986 MeV. For

these isotopes, the pkupn interaction was more in agreement with the available experimental data.

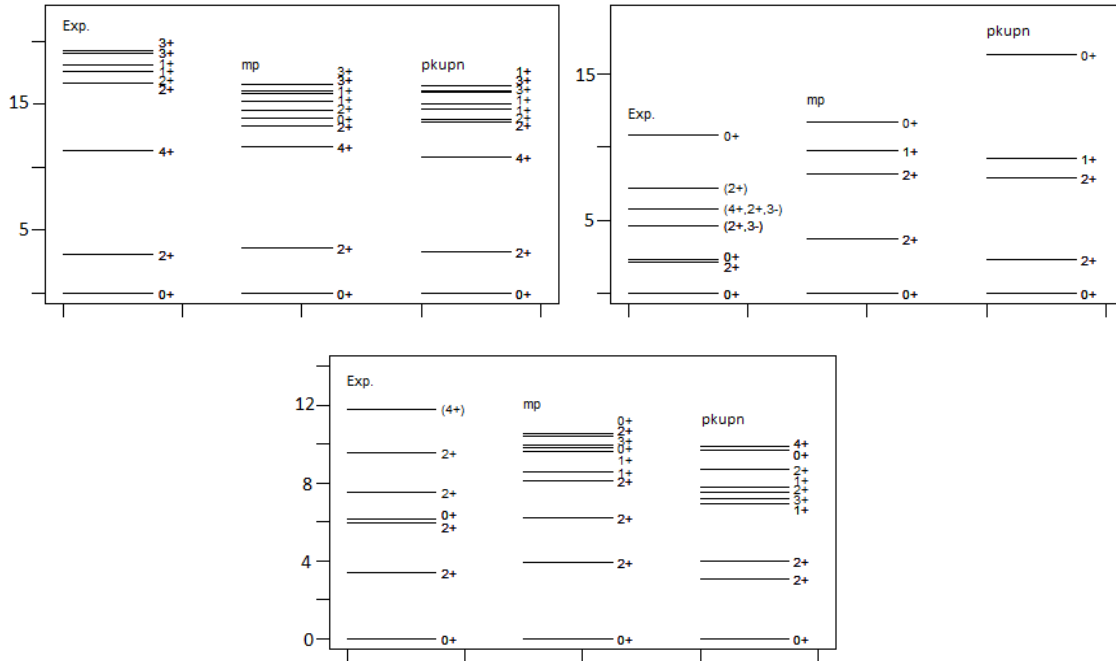
The results of the calculations with the double mass number Be isotopes are shown in Fig.4. The ground-state spin-parity value was calculated as  $0^+$ , in agreement with the experimental data. It was seen that the spin-parity sequences obtained from the theoretical calculations were almost the same as the experimental values. Only the spin of the fourth excited level in the mp interaction was included as  $0^+$ . The experimental energy spectrum for this isotope was  $2^+$  3.030 MeV,  $4^+$  11.350 MeV,  $2^+$  16.626 MeV,  $2^+$  16.922 MeV,  $1^+$  17.640 MeV,  $1^+$  18.150 MeV,  $3^+$  19.069 MeV, and  $3^+$  19.235 MeV. It has been observed that the sequence order in the calculations made with pkupn is also the same. Accordingly, the energy spectrum was  $2^+$  3.262 MeV,  $4^+$  10.814 MeV,  $2^+$  13.623 MeV,  $2^+$  13.734 MeV,  $1^+$  14.657 MeV,  $1^+$  14.996 MeV,  $3^+$  15.932 MeV,  $3^+$  16.028 MeV and  $1^+$  16.443 MeV. In addition, according to the results obtained from the mp interaction, the spectrum was appeared to be  $2^+$  3.580 MeV,  $4^+$  11.604 MeV,  $2^+$  13.225 MeV,  $0^+$  13.905 MeV,  $2^+$  14.521 MeV,  $1^+$  15.260 MeV,  $1^+$  15.795 MeV,  $3^+$  16.078 MeV and  $3^+$  16.539 MeV.

In the calculations for  $^{10}\text{Be}$ , the ground-state spin-parity value was calculated as  $0^+$  in accordance with the experimental data. It was seen that the experimental excited states were  $2^+$  3.368 MeV,  $2^+$  5.958 MeV,  $0^+$  6.179 MeV,  $2^+$  7.542 MeV,  $2^+$  9.560 MeV and  $(4^+)$  11.760 MeV. Calculations with the mp interaction revealed that two  $1^+$  levels and one  $3^+$  level emerged. Accordingly, the excited states were calculated as  $2^+$  3.917 MeV,  $2^+$  6.186 MeV,  $2^+$  8.101 MeV,  $1^+$  8.660 MeV,  $1^+$  9.632 MeV,  $0^+$  9.789 MeV,  $3^+$  9.934 MeV,  $2^+$  10.436 MeV and  $0^+$  10.539 MeV. With the pkupn interaction, it was observed that two  $1^+$  and one  $3^+$  levels emerged, and the ambiguous  $4^+$  level was also supported in the calculations. Excited states from mp interaction were in the form of  $2^+$  3.051 MeV,  $2^+$  4.012 MeV,  $1^+$  6.958 MeV,  $3^+$  7.185 MeV,  $2^+$  7.513 MeV,  $1^+$  7.783 MeV,  $2^+$  8.702 MeV,  $0^+$  9.701 MeV and  $4^+$  9.876 MeV.

Finally, when the calculations for  $^{12}\text{Be}$  were examined, it was seen that the ground-state spin-parity was calculated as  $0^+$ , which was in agreement with the experimental data. Experimental first and second excited energies were  $2^+$  2.109 MeV and  $0^+$  2.251 MeV. According to the calculations made with mp and pkupn interactions, these levels were  $2^+$  3.707 MeV and  $2^+$  8.201 MeV for mp,  $2^+$  2.328 MeV and  $2^+$  7.920 MeV for pkupn. There was no clarity in the majority of spin-parity values of subsequent experimental excited states. These levels were  $(2^+, 3^-)$  4.580 MeV,  $(4^+, 2^+, 3^-)$  5.724 MeV,  $(2^+)$  7.200 MeV and  $0^+$  10.800 MeV, respectively. According to the mp interaction, the third and fourth excited levels were

obtained as  $1^+ 9.727$  MeV and  $0^+ 11.704$  MeV. However, with pkupn, these values were calculated as  $1^+ 9.193$  MeV and  $0^+ 16.396$  MeV.

For these isotopes, the pkupn interaction was more in agreement with the available experimental data.



**Fig.4** The energy spectra for  $^8\text{Be}$  (top-left),  $^{10}\text{Be}$  (top-right) and  $^{12}\text{Be}$  (bottom) isotopes from literature experimental data and the results from shell-model calculations with mp and pkupn interactions

#### 4. Conclusion

In this study, the nuclear excited states of He and Be nuclei on the  $^4\text{He}$  core were investigated in the p-shell model space. Different two-body interaction have been considered for the calculations. The results of two of them which gave better results were presented. These interactions were ckihe and pkba for He isotopes and mp and pkupn for Be isotopes. For the  $^{5,7}\text{He}$  isotope, almost no experimental data were available in the literature. For  $^9\text{He}$ , however, spin-parity values were not determined. For the  $^{6,8}\text{He}$  isotopes, the spin-parity values of the excited states were either undetermined or ambiguous. For  $^{9,11,12}\text{Be}$  isotopes, there was ambiguity in spin-parity values of many levels. With the calculations carried out in this study, suggestions were made to the uncertainties in spin-parity and the energies of the levels without experimental data in the literature were calculated. It was observed that the interaction of pkba for He isotopes and pkupn for Be isotopes gave generally more consistent results with the experimental data. In the pkba and pkupn interactions, the two-body matrix elements are from Paris interaction. The single-particle energies are from least-squares fit to the 85 levels of the nuclei with mass number between 5 and 16. The single-particle energies are 3.8282 MeV and 1.7440 MeV for  $p_{1/2}$  and  $p_{3/2}$  levels.

The total numbers of two-body matrix elements are 15 and 44 for pkba and pkupn interactions, respectively.

#### Conflict of Interest

The authors have no conflict of interest.

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