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Estimation of Inelastic Longitudinal Electron Scattering Form Factors in $^{58,60}\text{Ni}$ Nuclei Using OXBASH Code

D. Murshedi*, A. D. Salman

ORCID: 0000-0002-6564-1744, 0000-0002-9443-9297

Department of Physics, College of Science, Karbala University, Karbala, Iraq

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ABSTRACT

Many effective interactions have been developed to provide a better understanding of the nuclear properties by starting with the realistic nucleon-nucleon (NN) interaction and using quantum-mechanical many-body theory. In this study, the calculations of inelastic longitudinal electron scattering form factors for C2 and C4 transitions have been examined in $^{58,60}\text{Ni}$ isotopes. The F5PVH effective interaction for the fp-shell is used with the nucleon-nucleon realistic interaction Michigan three-range Yukawa and Modified surface delta interaction as a two-body interaction. Two shell model codes, CP and OXBASH for windows, have been utilized in order to obtain the results. The core polarization effects are considered as the first-order perturbation theory with the effective charge of both proton and neutron. Based on the obtained results, the effective charge along with the core effects have significantly improved the calculation in term of the agreement with the experimental data. Also, the ^{60}Ni nuclei tend to have a better agreement when compared to ^{58}Ni nuclei.

Keywords: Inelastic scattering, Core polarization, Form factor, Ni Nuclei, OXBASH code

1. Introduction

Electron scattering from nuclei is assumed to be a practical method in the examination of nuclear structure. Numerous microscopic and macroscopic theories have been implemented to explore the excitation in nuclei. There are two types of electron scattering; elastic and inelastic scattering. In the first type, the nucleus is remained in the ground state, in which the static properties such as the static distribution of charge and magnetization can be measured. While in the second type, the nucleus is set to be in an excitation state from which the measurement of nuclear dynamical properties, such as the transition densities and current densities can be estimated [1-3].

Among the different nuclear models that are utilized to depict the nuclear structure, the shell model is addressed to be the most appropriate model to express the behavior of the nuclei [4]. There have been numerous studies that

have shown the success of the shell model is related to the choice of effective interaction. In other words, the two-body effective interaction is essential to ensure the success of the nuclear shell model, which measures the accuracy of the shell-model calculations [5]. Some studies have tested the accuracy of the shell model using F5PVH interaction. The F5PVH effective interaction is used to determine C2 and C4 form factors in the ^{65}Cu nucleus. When the core polarization effects have been ignored, it is shown that the estimated form factors were not well compatible with experimental data. While the consideration of core polarization effects generated a significant improvement compared to the experimental data [6]. Salman et al. [7] reported that core polarization effects using the F5P shell model with the F5PVH effective interaction in the $^{64,66,68}\text{Zn}$ isotopes have also enhanced the results in comparison with experimental results. Jassim and Faris [8] calculated the structure of $^{58,62}\text{Ni}$ nuclei utilizing the F5PVH interaction. The

*Corresponding author.

E-mail address: doaa.r244@gmail.com, (D. Murshedi)

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obtained results indicate that very good agreements were achieved from C2 form factor and Charge Density Distribution compared with the experimental data.

In this paper, the Coulomb form factors for ^{58,60}Ni nuclei have been examined for two cases. In the first case, the core-polarization effects are considered through particle-hole excitation up to 6ħω. For the second case, the core-polarization effects are neglected. Also, the effective charge for the proton and neutron for the two cases is included in the determination of the inelastic longitudinal electron scattering form factors.

2. OXBASH Code

OXBASH, which is created by brown, is a set of codes for carrying out shell-model calculations with dimensions up to about 50,000 in the J-T scheme and about 2,000,000 in the M-scheme [9]. It is used for estimating the energy levels of hundreds of nuclei and validate the results by comparing them with those acquired by experiments. Moreover, calculation of charge density, charge radii and form factor can be obtained with the aid of OXBASH code [10]. Energies of Potassium isotopes (A=38-40) was computed using the OXBASH code by Mohammadi et al. They compared the calculated energy states with the experimental data, the obtained results had a good agreement with the experimental data [11]. Also, using OXBASH code, Majeed and Hussain calculated inelastic electron scattering form factors for fp-shell nuclei ⁴²Ca, ⁴⁴Ca, ⁴⁶Ti, ⁴⁸Ti, ⁵⁰Cr and ⁵⁴Fe using GXPF1 and FPD6 effective interactions. They concluded that the CP effects are found to be crucial in the estimations of the C2 and C4 form factors and presents notably good agreement over the fp-shell model calculations [12].

In another study carried out by Selman et al. [13], the form factor for inelastic electron scattering to 2+ and 4+ states in ^{50, 52, 54}Cr have been examined by the use of the shell model. Longitudinal C2 and C4 multipolarity are studied for these states. Core polarization effects are involved through the first-order perturbation theory and the matrix elements are estimated with Modified surface delta Interaction (MSDI). The addition of core polarization resulted in an improvement of the calculated form factor, providing good agreement with experimental values.

3. Theoretical method

3.1 Estimation of the inelastic longitudinal electron scattering form factors

In order to estimate the inelastic longitudinal electron scattering form factors involving angular momentum and momentum transfer q, the following equations which have been adopted from the literature have been used [14].

$$|F_J(q, \theta)|^2 = \frac{4\pi}{z^{2(2J_i+1)}} |\langle J_f || \hat{T}_\Lambda^\eta(q) || J_i \rangle|^2 |F_{cm}(q)|^2 |F_{fs}(q)|^2 \quad (1)$$

where F_{cm} is represent the center of mass correction and this correction can be described as [15].

$$F_{cm}(q) = e^{\frac{q^2 b^2}{4A}} \quad (2)$$

where b is the harmonic- oscillator size parameter, A is the nuclear mass number and q is the momentum transfer.

One more correction to the form factor formula is the nucleon finite-size correction, which takes into consideration the finite size of the nucleon and is presented as [16].

$$F_{fs}(q) = [1 + (q/4.33fm^{-1})^2]^{-2} \quad (3)$$

The reduced matrix elements in spin and isospin space of the longitudinal operator between the final and initial for many particles states of the system including the configuration mixing are given in terms of the One Body Density Matrix (OBDM) elements times the single-particle-matrix elements of the longitudinal operator [17].

$$\langle f || \hat{T}_{JT}^L || i \rangle = \sum_{a,b} OBDM^{JT}(i, f, a, b) \langle b || \hat{T}_{JT}^L || a \rangle \quad (4)$$

3.2 Core-Polarization Effects

The core polarizations are included through microscopic theory, which combines shell- model wave functions and configurations with higher energy as first-order perturbations. The reduced matrix elements of the electron scattering operator consist of two parts; one is for the "Model space" matrix elements, and the other is for the "core-polarization" matrix elements [18].

$$\langle \Gamma_f || \hat{T}_\Lambda^\eta || \Gamma_i \rangle = \langle \Gamma_f || \hat{T}_\Lambda^\eta || \Gamma_i \rangle_{ms} + \langle \Gamma_f || \delta \hat{T}_\Lambda^\eta || \Gamma_i \rangle_{cp} \quad (5)$$

where η represents the longitudinal (L), or transverse form factors (electric (E) and magnetic (M)). The model space matrix elements can be written as:

$$\langle \Gamma_f || \hat{T}_\Lambda^\eta || \Gamma_i \rangle_{ms} = \sum_{\alpha, \beta} OBDM(\Gamma_i, \Gamma_f, \alpha, \beta) \langle \alpha || \hat{T}_\Lambda^\eta || \beta \rangle_{fp} \quad (6)$$

The OBDM is determined from OXBASH code [9], the core polarization matrix elements are stated as:

$$\langle \Gamma_f || \delta \hat{T}_\Lambda^\eta || \Gamma_i \rangle_{cp} = \sum_{\alpha, \beta} OBDM(\Gamma_i, \Gamma_f, \alpha, \beta) \langle \alpha || \delta \hat{T}_\Lambda^\eta || \beta \rangle_{cp} \quad (7)$$

Based on the first-order perturbation theory, the singleparticle core-polarization term is presented by [19].

$$\langle \alpha | \delta \hat{T}_j^\eta | \beta \rangle = \langle \alpha | \hat{T}_j^\eta \frac{Q}{E-H^{(0)}} V_{res} | \beta \rangle + \langle \alpha | V_{res} \frac{Q}{E-H^{(0)}} \hat{T}_j^\eta | \beta \rangle \quad (8)$$

Where the operator Q is the projection operator onto space outside the model space. The single-particle core-polarization terms shown in equation (8) are written as [20]:

where the index α_1 runs over particle states and α_2 over hole states and e is the single-particle energy, and is determined as [15]:

$$\langle \alpha | \hat{T}_\Lambda^\eta \frac{Q}{E-H^{(0)}} V_{res} | \beta \rangle = \sum_{\alpha_1 \alpha_2 \Gamma} \frac{(-1)^{\beta+\alpha_2+\Gamma}}{e_{\beta^-} - e_{\alpha_1^-} - e_{\alpha_1^+} + e_{\alpha_2^+}} (2\Gamma+1) \begin{Bmatrix} \alpha & \beta & \Lambda \\ \alpha_2 & \alpha_1 & \Gamma \end{Bmatrix} \langle \alpha \alpha_1 | V_{res} | \beta \alpha_2 \rangle \times \langle \alpha_2 | \hat{T}_\Lambda^\eta | \alpha_1 \rangle \sqrt{(1 + \delta_{\alpha_1 \alpha}) (1 + \delta_{\alpha_2 \beta})} \quad (9)$$

+term with α_1 and α_2 exchange with an over minus sign

$$e_{nlj} = \left(2n + l - \frac{1}{2} \right) \hbar \omega + \begin{cases} -\frac{1}{2}(l+1) \langle f(r) \rangle_{nl} & \text{for } j = l - \frac{1}{2} \\ \frac{1}{2} l \langle f(r) \rangle_{nl} & \text{for } j = l + \frac{1}{2} \end{cases} \quad (10)$$

$$\text{With } \langle f(r) \rangle_{nl} \approx -20A^{-\frac{2}{3}} \text{ and } \hbar \omega = 45A^{-\frac{1}{3}} - 25A^{-\frac{2}{3}} \quad (11)$$

For the residual two-body interaction (V_{res}), MSDI and M3Y interaction of Bertsch et al. [21] are adopted.

3.3 Electromagnetic Transition Strength

The electromagnetic transition probability is defined at the photon point, where the momentum transfer $q = k = E_x/\hbar c$ where E_x is the excitation energy. The form factor at ($q = k$) [22] is:

$$|F_f^L(k)|^2 = \frac{4\pi}{(2J_i+1)Z^2} \left| \int_0^\infty dr r^2 j_J(kr) \rho_J(i, f, r) \right|^2 \quad (12)$$

$$j_J(kr) = \frac{(kr)^J}{(2J+1)!!} \left(1 - \frac{1}{2} \frac{(kr)^2}{z(2J+3)} + \dots \right) \quad (13)$$

Retaining only the leading term in the series expansion of $j_J(kr)$, one obtains:

$$j_J(kr) \cong \frac{(kr)^J}{(2J+1)!!} \quad (14)$$

Then equation (12) becomes:

$$|F_f^L(k)|^2 = \frac{4\pi}{(2J_i+1)Z^2} \left[\frac{k^J}{(2J+1)!!} \right] \left| \int_0^\infty dr r^{J+2} \rho_J(i, f, r) \right|^2 \quad (15)$$

The multipole matrix element is defined as [20]:

$$Q_{ft}^J = \int_0^\infty dr r^{J+2} \rho_J(i, f, r) \quad (16)$$

And the reduced transition probability is defined as:

$$B(CJ) = \frac{|Q_{ft}^J|^2}{2J_i+1} \quad (17)$$

Then the reduced transition probability B(CJ) is written in terms of the form factor in the limit ($q=k$) photon point as:

$$B(CJ) = \frac{[(2J+1)!!]^2 Z^2 e^2}{4\pi k^2 J} |F_f^L(k)|^2 \quad (18)$$

4. Results and discussion

4.1 ⁵⁸Ni nucleus

Two particles are distributed over the $1f_{5/2}, 2p_{3/2}, 2p_{1/2}$ orbits with ⁵⁶Ni as the core. The single-particle radial wave function used are those of harmonic oscillator potential with size parameter $b=1.96 \text{ fm}$ [7]. The OBDM obtained from OXBASH code using the F5PVH interaction.

4.1.1 The (2_1^+ 1) state at 1.454 MeV

The ⁵⁸Ni nucleus is excited from the ground state $j_i^\pi T_i = 0^+ 1$ to the state $j_i^\pi T_i = 2^+ 1$ with the excitation energy of 1.454 MeV. As presented in Table 1, the experimental reduced transition probability B(C2 \uparrow) is equal to $695 \pm 20 \text{ e}^2 \text{ fm}^4$ [23], while the theoretically calculated reduced transition probability is equal to $23.199 \text{ e}^2 \text{ fm}^4$, which is low in comparison to the estimated values. As the core polarization effects are included, the B(C2) value will be equal to $538.2 \text{ e}^2 \text{ fm}^4$, which is lower than the experimental values about 29%, this is because of the effect of adjustable parameters of MSDI which are adjusted for all nuclei under consideration.

The values of the one-body density matrix elements (OBDM) are shown in Table 2. Fig.1 displays the C2 transition using MSDI interaction. The core polarization noticed to be decisive to estimate the exact shape of the form factor (the dashed dot curve), the underestimation of the curve can be solved by introducing an effective charge (the solid curve) to the proton and neutron. It has been seen that the calculations including effective charges are creating a noticeable agreement with the experimental values in the first and third peaks, but the second peak is underestimated. On the other hand, as exhibited in Fig.2, the M3Y interaction failed to describe the shape of the form factor.

Table 1. The values of the reduced transition probabilities B(C2) (in unit e2.fm4) for The ^{58}Ni nucleus in comparison with the experimental data

<i>Nucleus</i>	J_i^π	T_i	J_f^π	T_f	$E_x(\text{MeV})$	fp	$fp+Cp$	<i>Exp.</i>
^{58}Ni	0^+	1	2^+	1	1.454	23.199	538.2	695 ± 20 [23]

Table 2. The OBDM values for C2 transition in ^{58}Ni

$^{58}\text{Ni} (E_x=1.454)$		C2	
J_i	J_f	OBDM ($\Delta T=0$)	OBDM ($\Delta T=1$)
$3/2$	$3/2$	-0.13643	-0.11140
$3/2$	$5/2$	0.13559	0.11071
$3/2$	$1/2$	-0.13677	-0.11167
$5/2$	$3/2$	-0.07833	-0.06396
$5/2$	$5/2$	-0.65956	-0.53852
$5/2$	$1/2$	-0.19730	-0.16110
$1/2$	$3/2$	-0.14353	-0.11719
$1/2$	$5/2$	0.35839	0.29263

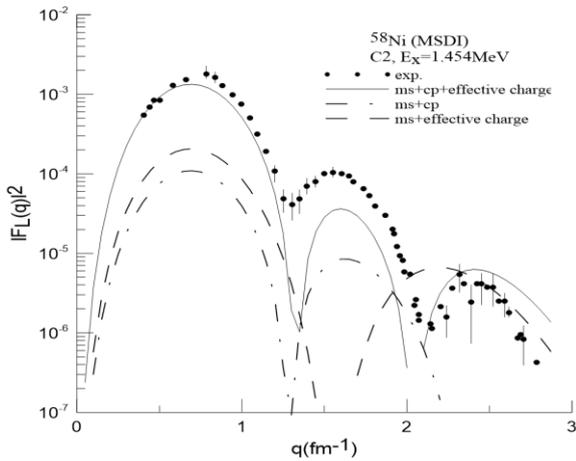


Fig. 1 Inelastic longitudinal form factors for the transition to the 2_1^+ state in the ^{58}Ni nucleus using MSDI interaction, the experimental values are extracted from Ref. [24]

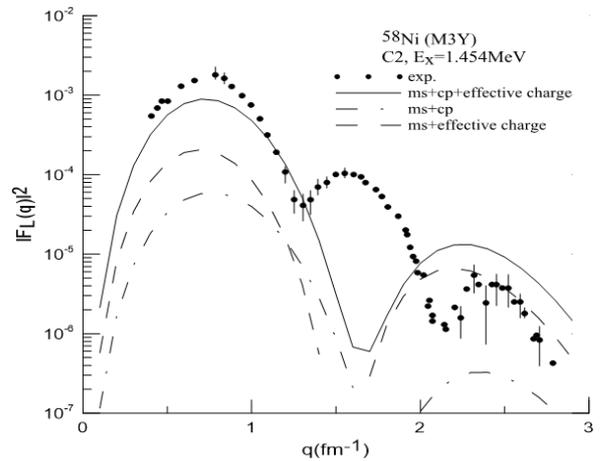


Fig. 2 Inelastic longitudinal form factors for the transition to the 2_1^+ state in the ^{58}Ni nucleus using M3Y interaction, the experimental values are extracted from Ref. [24]

4.2 ⁶⁰Ni nucleus

For this nucleus, ⁵⁶Ni is assumed as inert core leaving four valance nucleons distributed over 2p-1f shell model space. The single-particle wave functions for all considered transitions are of the harmonic oscillator potential with size parameter $b_{rms}=1.97 \text{ fm}$ [7]. The OBDM was also obtained from the OXBASH code using the F5PVH interaction.

The values of the one-body density matrix elements (OBDM) are shown in Table 4 and Table 6. Fig.3 displays the C2 transition using MSDI interaction. The core polarization was considered to be fundemenatl for the estimation of exact shape of the form factor (the dashed dot curve), the underestimation of the curve can be solved by introducing an effective charge (the solid curve) to the proton and neutron. It has been seen that the inclusion of the effective charges developed an obvious agreement in comparison with the experimental values in the first and third peaks, while the second peak is noted to be underestimated. On the other hand, as illustrated in Fig.4, the M3Y interaction failed to describe the shape of the form factor.

4.2.1 The (2_1^+ 2) state at 1.33 MeV

The nucleus is excited from the ground- state $j_i^\pi T_i = 0^+ 2$ by the incident electron to the state $j_f^\pi T_f = 2_1^+ 2$

with excitation energy of 1.33MeV. As demonstrated in Table 3, the estimated B(C2) value ignoring the core-polarization effects is equal to $31.354 \text{ e}^2\text{fm}^4$, which is low in comparison with the measured value to $933 \pm 15 \text{ e}^2\text{fm}^4$ [23].

As the core polarization effects considered, the estimated B(C2) value has found to be equal to $983.1 \text{ e}^2\text{fm}^4$. In other words, the inclusion of the core polarization effects enhanced the agreement of the measured values compared with the experimental data. The values of the one-body density matrix elements (OBDM) are presented in Table 4. The coulomb C2 form factor for the ⁶⁰Ni nucleus is exhibited in Fig.3. It is observed that involving the model space and the effective charge underestimated the experimental values while the theoretical calculation enhanced when the core polarization (cp) effect is included with the effective charge (the solid curve). The first and third peaks are reasonably well reproduced. However, the second peak underestimates the experimental values. Furthermore, the calculated form factor with cp exhibited a good agreement with experimental values which is clearly noticed in the obtained shape as well as the diffraction minimum at the correct momentum transfer.

Table 3. The values of the reduced transition probabilities B(C2) (in unit $\text{e}^2.\text{fm}^4$) for the ⁶⁰Ni nucleus in comparison with the experimental data

Nucleus	J_i^π	T_i	J_f^π	T_f	Ex(MeV)	f_p	f_{p+Cp}	Exp.
⁶⁰ Ni	0^+	2	2^+	2	1.33	31.354	983.1	933 ± 15 [23]

Table 4. The OBDM values for the C2 transition in ⁶⁰Ni.

⁶⁰ Ni ($E_x=1.33$)		C2	
J_i	J_f	OBDM($\Delta T=0$)	OBDM($\Delta T=1$)
3/2	3/2	-0.41705	-0.29490
3/2	5/2	0.21391	0.15126
3/2	1/2	-0.34433	-0.24348
5/2	3/2	-0.14066	-0.09946
5/2	5/2	-0.68336	-0.48321
5/2	1/2	-0.34200	-0.24183
1/2	3/2	-0.37332	-0.26398
1/2	5/2	0.56072	0.39649

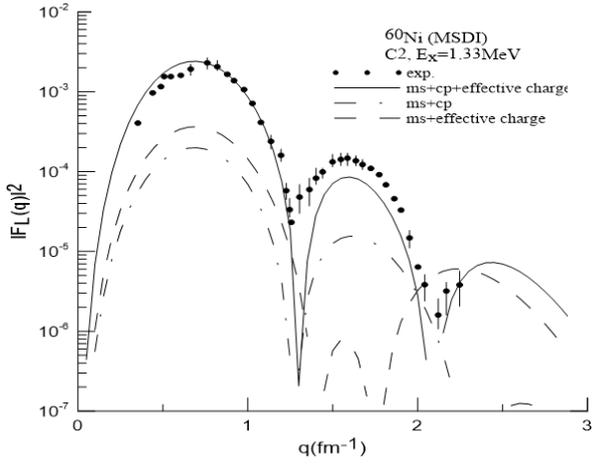


Fig. 3 Inelastic longitudinal form factors for the transition to the 2_1^+ state in the ^{60}Ni nucleus using MSDI interaction, the experimental values are extracted from Ref. [24]

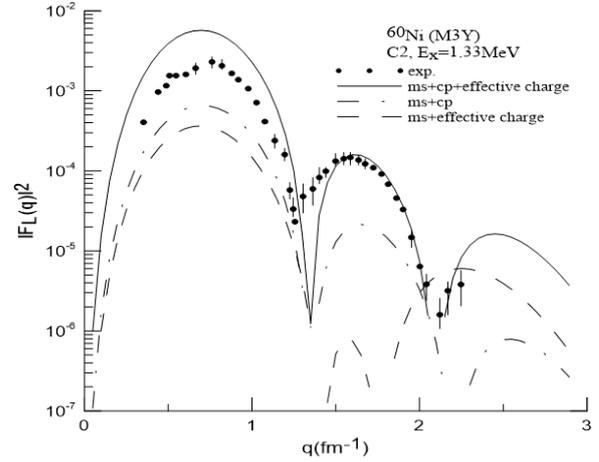


Fig. 4 Inelastic longitudinal form factors for the transition to the 2_1^+ state in the ^{60}Ni nucleus using M3Y interaction, the experimental values are extracted from Ref. [24]

4.2.2 The (4_1^+ 2) state at 2.5 MeV

The nucleus is excited from the ground- state $J_i^\pi T_i = 0^+ 2$ by the incident electron to the state $J_f^\pi T_f = 4_1^+ 2$ with excitation energy of 2.5MeV.

As indicated in Table 5, the obtained B(C4) value ignoring core polarization effects is $431.766 e^4\text{fm}^8$, which is low compared with the measured value $1.50\pm 0.3E+05 e^4\text{fm}^8$ [25]. When the core-polarization effects are considered, the B(C4) value is approximately equal to $1.864E+05e^4\text{fm}^8$ which is reasonably accepted taking into account the standard deviation.

The values of the one-body density matrix elements (OBDM) are shown in Table 6. Fig.5 depicts the C4 transition using MSDI interaction. It was observed that the experimental data of Ref. [24] are described quite well after the inclusion of the core polarization together with the effective charge. For the M3Y interaction, Fig.6, the model space is underestimated, the inclusion of the core polarization and the effective charge enhance the results. This improvement brings the form factors very close to the experimental data.

Table 5. The values of the reduced transition probabilities B(C4) (in unit $e^2.\text{fm}^8$) for The ^{60}Ni nucleus in comparison with the experimental data

Nucleus	J_i^π	T_i	J_f^π	T_f	Ex(MeV)	fp	fp+Cp	Exp.
^{60}Ni	0^+	2	4^+	2	2.5	431.766	$1.864E+05$	$1.50\pm 0.3E+05$ [25]

Table 6. The OBDM values for the C4 transition in ^{60}Ni .

$^{60}\text{Ni} (E_x=2.5)$		C4	
J_i	J_f	OBDM ($\Delta T=0$)	OBDM ($\Delta T=1$)
3/2	3/2	0.30989	0.21912
3/2	5/2	-0.88507	-0.62584
5/2	3/2	0.58233	0.41177

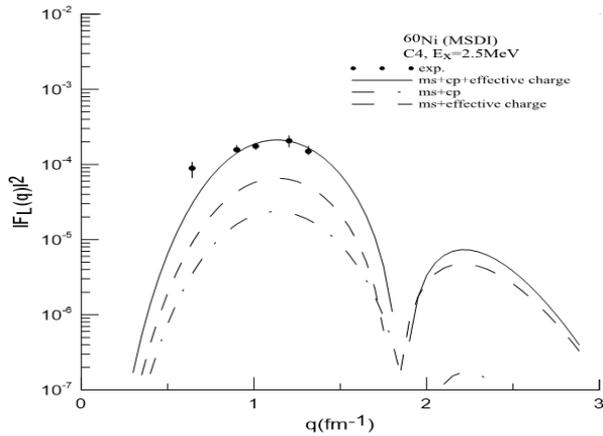


Fig. 5 Inelastic longitudinal form factors for the transition to the 4_1^+ state in the ^{60}Ni nucleus using MSDI interaction, the experimental values are extracted from Ref. [24]

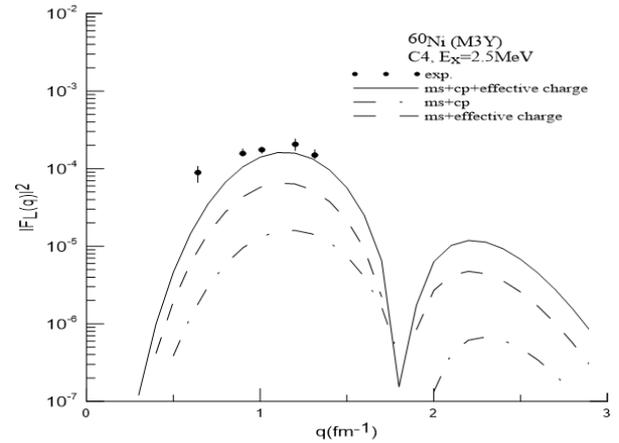


Fig. 6 Inelastic longitudinal form factors for the transition to the 4_1^+ state in the ^{60}Ni nucleus using M3Y interaction, the experimental values are extracted from Ref. [24]

5. Conclusion

In this study, F5PVH effective interaction for the fp-shell is used with the nucleon-nucleon realistic interaction Michigan three-range Yukawa and Modified surface delta interaction as a two-body interaction. Based on the obtained results, the following points can be drawn:

1. The C2 and C4 transitions are less successful when the fp-shell model is included and can be enhanced when the core polarization and the effective charges are taken into account.
2. The inclusion of the effective charges reduces the parameters of the MSDI to (0.4MeV).
3. The B(C2) and B(C4) values for ^{60}Ni are very close to estimated values.
4. The C2 transition is less successful when the fp-shell model is included and can be enhanced when the core polarization and the effective charges are taken into account.

Conflict of Interest

The authors have no conflict of interest.

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