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BISTATIC SONOBUOY DEPLOYMENT CONFIGURATION FOR STATIONARY TARGETS

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Abstract

Countering submerged targets using bistatic and multistatic sonobuoy systems is a fundamental problem in Anti-Submarine Warfare. A key question is: what is the best deployment geometry of sensors to successfully detect a submarine threat in a field of interest? The unique properties of these systems distinguish this problem from the conventional ones. This paper examines the optimum deployment strategies of bistatic sonobuoys against stationary or low speed targets.

SABİT HEDEFLERE KARŞI BİSTATİK SONOBOY YERLEŞİM KONFİGURASYONU

Özetçe

Bistatik ve multistatik (çoklu alıcılı vericili sonar sistemleri) sonoboy sistemleri ile sualtı hedeflerine karşı koruma sağlama denizaltı savunma harbinin temel problemlerinden birisidir. "İlgi alanımızdaki bir denizaltının başarılı bir şekilde tespiti için en iyi sensör yerleşim konfigürasyonu nasıl olmalıdır?" sorusu temel problemi ortaya koymaktadır. Söz konusu sistemlerin kendilerine has özellikleri bu yerleşim problemini diğer klasik yerleşim problemlerinden farklı yapmaktadır. Bu çalışma bistatik sonoboyların sabit veya düşük süratli hedeflere karşı en iyi yerleşim planının oluşturulmasını amaşlamaktadır.

Keywords: *Bistatic detection, probability of detection, sonobuoy.*
Anahtar Kelimeler: *Bistatik tespit, tespit olasılığı, sonoboy.*

1. INTRODUCTION

The basic operating concept of a bistatic sonar network is to emit sound energy from a source into the water and listen for the reflected echoes returning across the receiver to detect, localize and track targets of interest. The source of energy can be a ship with a hull-mounted sonar, a helicopter with a dipping sonar, an explosive charge dropped by an aircraft or an active sonobuoy. The receiver can be a passive sonar, a passive sonobuoy or a hydrophone system (Washburn, 2010).

In a monostatic system the source and receiver are co-located whereas in a bistatic system they are separated by a distance large enough to be comparable to the distance to the potential target. In other words, a bistatic active system is a generalization of the traditional monostatic active sonar to the case where the source and receiver are not co-located. A multistatic system consists of multiple sources and receivers – each source receiver couple forms a bistatic system - distributed over the surveillance area. (Krout et al, 2009).

For a certain environmental condition, the performance of a bistatic sonobuoy system is determined by its geometry and is characterized by Cassini ovals depending on the location of both source and receiver (Wang et al, 2008). The problem of devising optimal sensor configurations arises and it is significantly more complex than the problem in monostatic systems. In this study, we investigate configuration strategy of bistatic sonobuoys that are performing area search over a field of interest, *F*, and quantify the Probability of Detection (*PoD*) capability of such systems for stationary or low speed targets. These strategies can be used by designers to select key system characteristics (i.e. source level, receiver gain) as well as to plan the geometry of the bistatic systems (i.e. source and receiver locations). The accuracy of all proposed strategies is confirmed through detailed Monte Carlo simulations.

The organization of the paper is as follows. First bistatic detection criteria and basic properties of Cassini ovals are described in section 2.

Section 3 proposes the optimal sensor separation distances between bistatic sonobuoys. The comparison of analytical estimates with Monte Carlo simulation data is presented in section 4 and finally section 5 summarizes the main results.

2. BISTATIC DETECTION

In James & James (1992), a Cassini oval is defined as "*the locus of the vertex of a triangle when the product of the sides adjacent to the vertex is a constant and the length of the opposite side is fixed*". If we apply this definition to the bistatic triangle in Figure 1, the vertex is at the target, b^2 denotes the constant, R_1 and R_2 are the sides adjacent to the vertex and the separation distance, $2a$, between the source and receiver is the length of the opposite side. If the sensors are fixed at $(\pm a, 0)$, its Cartesian equation will be:

$$\left[(x-a)^2+y^2\right]\left[(x+a)^2+y^2\right]=b^4, \quad a,b\in\mathbb{R}. \tag{1}$$

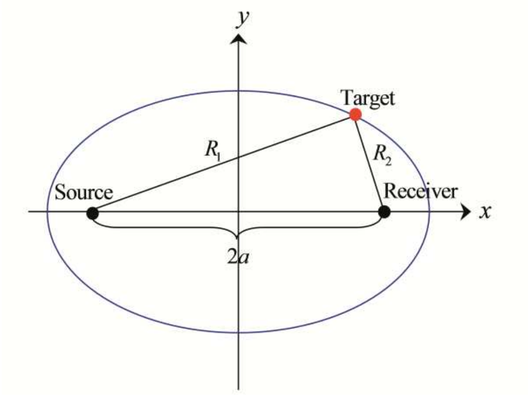


Figure 1. Bistatic triangle

Figure 2 from Washburn and Karatas (2015) illustrates the ovals for different values of a/b where b is fixed at 1 for simplicity. Interested reader can refer Karatas (2013) for details on Cassini ovals.

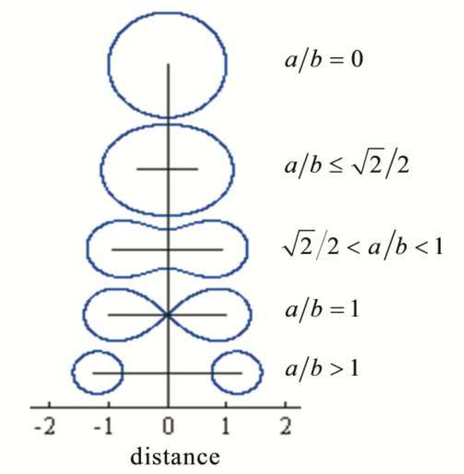


Figure 2. A family of Cassini ovals (b=1) (from Washburn and Karatas (2015))

To detect the performance of such systems computing the area of a Cassini oval is essential in the sense that maximizing the area coverage also maximizes *PoD* for stationary targets. The area of a Cassini oval, A_C, can be reduced to a single numerical integration as follows. Since the oval is symmetric with respect to both axes we can compute A_C by multiplying the area of a quarter oval by four.

$$A_C = \begin{cases} 4\int_0^{\sqrt{a^2+b^2}} f_C(x)dx & ,a/b \le 1 \\ 4\int_{\sqrt{a^2-b^2}}^{\sqrt{a^2+b^2}} f_C(x)dx & ,a/b > 1 \end{cases} \tag{2}$$

where $y = f_C(x) = \pm\sqrt{-a^2 - x^2 \pm \sqrt{4x^2a^2 + b^4}}$ after solving (1) for y. To compute A_C one can also use the below approximation derived by (Willis, 2005):

$$A_C \approx \begin{cases} \pi b^2 \left(1 - \dfrac{a^4}{4b^4} - \dfrac{3a^8}{64b^8}\right) & , a/b \le 1 \\ \dfrac{\pi b^4}{2a^2}\left(1 + \dfrac{b^4}{8a^4} + \dfrac{3b^8}{64a^8} + \dfrac{25b^{12}}{1024a^{12}}\right) & , a/b > 1 \end{cases} \tag{3}$$

Both the numerical integration (2) and the approximation (3) for A_C, normalized with respect to the monostatic area, πb^2, are plotted as a function of ratio *a/b* in Figure 3.

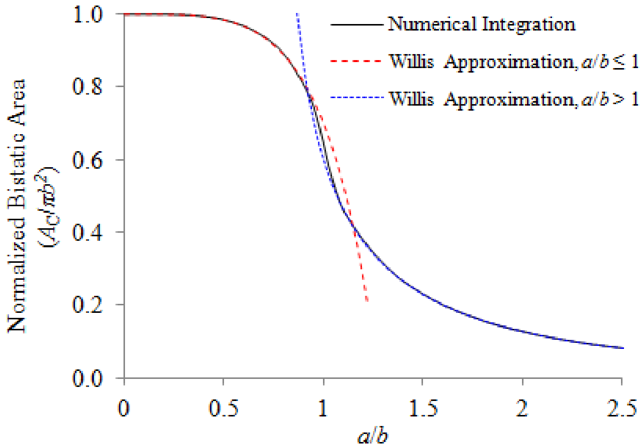


Figure 3. Normalized bistatic area computed with numerical integration and Willis approximation

3. OPTIMAL BISTATIC SONOBUOY DEPLOYMENT STRATEGY

A "bistatic sonobuoy couple" is a single source and receiver. Assume that a bistatic sonobuoy couple with a monostatic detection range of $b > 0$ is deployed within the field of interest $F \subset \mathbb{R}^2$ where F is a connected and closed convex set with area A_F. The distance between the sensors (separation distance) is $2a \geq 0$. The bistatic couple can only detect events within its sensing zone *C* a point in *F* is said to be covered if it is inside *C*.

We consider the scenario where a target is stationary and assumed to be uniformly distributed over *F*. This is a reasonable assumption for the cases where there is no priori knowledge about possible target location and the target is likely to be everywhere in the search area. If we assume that a target that falls into the sensing zone *C* for a certain source-receiver position, is detected with probability 1 and therefore *PoD* simply depends on the fraction of area covered, $PoD = A_C \setminus A_F$. To maximize *PoD* for a given A_F, we need to maximize A_C, by controlling the parameter *a* (semi-distance between buoys). Figure 3 shows that the bistatic area reaches its maximum value for $a/b = 0$ (when it is a regular circle) which implies that co-locating the source and receiver is the optimal strategy to maximize the *PoD* for stationary underwater targets. In this case, particularly since tactics are simplified when source and receiver are part of the same physical sonobuoy package, monostatic is better than bistatic.

Consider a special case where there are equal number of, say *m*, sources and receivers. There are two possibilities. In case A, we have multiple bistatic pairs where each receiver can hear exactly one source. In case B, we have a multistatic system with the same number of sources as receivers, but where a receiver can hear returns from any source. It is not difficult to conclude that case A would be better off being monostatic, at least when A_F is large enough to contain all the monostatic circles without overlap, since our argument can be repeated pair by pair. However this is not necessarily true for case B, the reason being that it is possible to get higher coverages by positioning them in different geometries such as the rectangular patterns as in Figure 4 which depicts some possible alternatives for m = 1, 2, 3, 6, and 10 source-receiver pairs. Interested reader can refer to Washburn and Karatas (2015) for details on creating optimal sonobuoy fields. After approximating the coverages by Monte Carlo simulations, the comparison of cases A and B in terms of coverage is depicted in Figure 5. The coverages increase linearly with *m* and multistatic case always performs better than the bistatic case due to the additional coverages for each receiver source coupling.

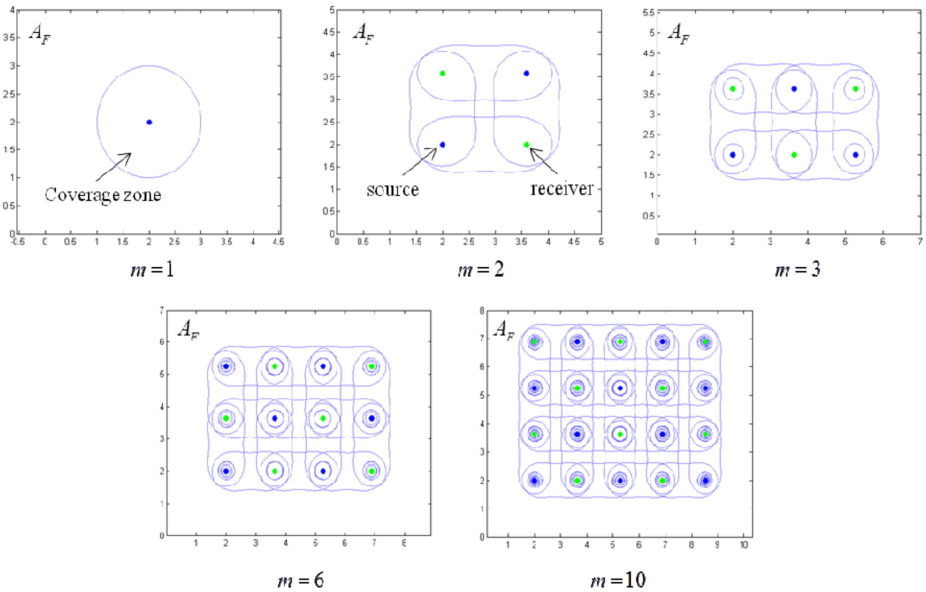


Figure 4. Coverage of equal number of multistatic sources and receivers deployed in rectangular patterns (*b* is assumed to be 1 for simplicity)

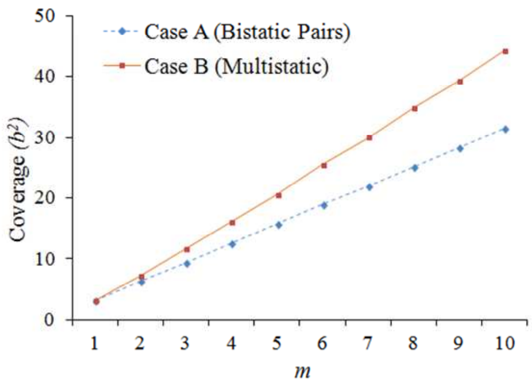


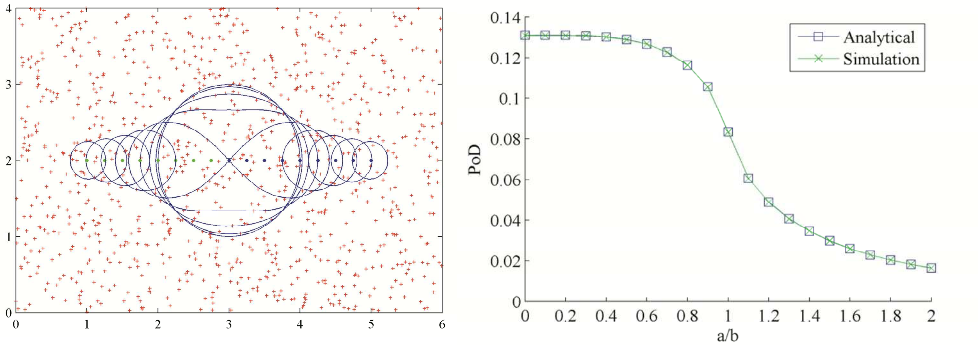
Figure 5. Coverage values for cases A and B.

4. SIMULATION

In this section we present the results of MATLAB® simulations to verify the theoretical results of *PoD* for stationary targets. Without loss of generality in all simulation runs we take b = 1 km.

In our experiments the targets are assumed to be randomly and uniformly distributed within region F of size 4x6 km. The distribution is "uniform" because in any reasonably large fraction of the area there are about the same number of targets as in any other fraction of the same size. The distribution is "random" because the exact location of each target is chosen at random to avoid producing a bias favoring one portion of the area over another. Since the placement and orientation of sonobuoys with respect to F do not affect A_C, we initially deploy both the source and receiver at the center of F such that the separation distance between sonobuoys is $2a = 0$ km. For each simulation run the distance is increased by increments of 0.02 km by pulling them apart from each other towards the opposite sides of F as seen in Figure 6(*i*). We generate 10^6 targets at the beginning of simulation and measure the number of targets that lie inside C for each a/b value. Consistent with our conclusion, *PoD* increases with A_C, and A_C attains its maximum value when $a/b = 0$.

Figure 6(*i*) illustrates a screenshot of the simulation process where the ovals are the bistatic detection zones for a series of source and receiver positions and the (+) marks are the target positions. Figure 6(*ii*) shows the theoretical values and simulation results of *PoD* between a/b = [0,2]. We observe an almost exact match between the theory and the simulation, conforming that the probability of detecting stationary targets within bistatic sensors is maximum when the separation distance between buoys is zero. Even though the $a/b = 0$ is optimal any ration a/b = [0, 0.5] can be conferred as a compromising solution since it is nearly optimal in all cases. Setting the ratio > 0 results in a bistatic field where sources and receivers are not at the same location (or not close to each other). This enables the decision makers to take advantage of the covertness of the receive platforms, which will make taking countermeasures difficult for the target.



(i) *(ii)*

Figure 6. (*i*) Source and receiver positions for some simulation steps and random target positions (*ii*) *PoD* values with respect to *a/b* values for A_F =24 km^2

5. CONCLUSION

In this paper, the problem of determining placement of a bistatic sonobuoy system for maximizing the target detection probability is considered. Assuming that the geometrical model of a bistatic system is a Cassini oval with a sensor separation distance of *2a* and equivalent monostatic detection range of *b*, the problem is to select the optimal *a/b* ratio so as to maximize *PoD.* For the stationary target scenario, *PoD* simply depends on the fraction of the area covered and *a/b* = [0, 0.5] is a good compromise which enables high coverage and also permits the covertness of the receiver. The accuracy of the analytical result is tested with computationally cumbersome Monte Carlo simulations and results show good agreement between the theory and simulation.

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